REPORT *of*

RESEARCH ACTIVITIES

July 1, 1964 — June 30, 1967

COWLES FOUNDATION

FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

PURPOSE AND ORIGIN

THE COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY, *established as an activity of the Department of Economics in 1955, has as its purpose the conduct and encouragement of research in economics, finance, commerce, industry, and technology, including problems of the organization of these activities. The Cowles Foundation seeks to foster the development of logical, mathematical, and statistical methods of analysis for application in economics and related social sciences. The professional research staff are, as a rule, faculty members with appointments and teaching responsibilities in the Department of Economics and other departments.*

The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs, Colorado. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. In 1955 the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation.

COWLES FOUNDATION FOR RESEARCH
IN ECONOMICS AT YALE UNIVERSITY

OFFICERS AND STAFF, 1964-67

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Alfred Cowles

Irving Fisher

NOTE ON REFERENCES TO PUBLICATIONS

The following abbreviations are used throughout this report in referring to publications or working papers of the Cowles Foundation:

IRVING FISHER, 1867–1947,
AND THE COWLES FOUNDATION

In this centennial year of the birth of Irving Fisher, it is appropriate to recall the various links between Fisher's thought and action as an economist and the origins and activities of the Cowles Commission (now Cowles Foundation) for Research in Economics.

The most important link is intellectual. It is the great influence of Irving Fisher's economic thought in the entire range of topics of research activity at the Cowles Foundation. As one example, the strong revival of capital theory of the last fifteen years builds directly on Fisher's fundamental and classic work, *The Theory of Interest*, and on other writings by him. Likewise, his work on index numbers and on distributed lags are landmarks in the development of econometric methods. The extent to which Irving Fisher's thought is alive in current economic research is illustrated by a forthcoming collective work, "Ten Economic Studies in the Tradition of Irving Fisher." One of these studies is an evaluation of Fisher's work by Paul A. Samuelson of the Massachusetts Institute of Technology. The other nine are all contributed by members of the Department of Economics of Yale University: Professors Fellner, Hall, Koopmans, Miller, Nerlove, Ruggles, Scarf, Tobin and Wallich. Of these, one is an account of Fisher as a scholar and individual. The other eight are contributions to the various fields of research in which Fisher was active. Four of the latter, written by members of the staff of the Cowles Foundation, are described below in the report on research activities.

The second link is historical. Fisher played an active role in the setting up of the Cowles Commission for Research in Economics, the predecessor of the Cowles Foundation. A full account of this history was given by Carl Christ in "Economic Theory and Measurement," A Twenty Year Research Report, 1932–52, of the Cowles Commission. Briefly, in 1931 and 1932, Fisher, Charles Roos, and Ragnar Frisch, in their capacities as President, Secretary, and leading European member, respectively, of the newly founded Econometric Society, discussed with Alfred Cowles his offer to set up and finance the Cowles Commission as a research organization in econometrics, to operate under the auspices and guidance of the Econometric Society, and recommended its acceptance to the Council of that Society. In this fusion

of scholarly evaluation and enlightened financial sponsorship the Cowles Commission, later Foundation, found its origin.

The third link is that Fisher was associated with Yale University for most of his professional career, 1890–1935, first in the Department of Mathematics, then in the Department of Political and Social Science, and ultimately in the Department of Economics. Thus the Cowles Foundation is now contributing to the continuation at Yale of the approach to economic theory and measurement so brilliantly initiated and represented by Irving Fisher.

INTERLOCKING RESEARCH TOPICS:
REPORT ON RESEARCH ACTIVITIES, 1964–1967

The Cowles Foundation is a symbiosis of individual scholars pursuing interrelated research in economics, with occasional forays into other social or political sciences.

Although the members of the staff differ widely in their outlook and interests, they have in common the use of quantitative, statistical, and mathematical approaches to economics and to social science. Moreover, although specific research topics may be problem-oriented or technique-oriented, in all work the motivation of adding to cumulatively useful knowledge is at least as important as that of contributing to the solution of a particular problem. In that sense the organization is devoted to basic research. Within this common orientation, the research interests of the staff are balanced between theory, policy, empirical study, and the methods of such study. The choice of research topic is an individual prerogative. The drift of the research of the group as a whole is therefore determined primarily by balanced selection in appointments.

Both the composition of the group and the free interaction of its members have produced a remarkable complementarity and meaningful diversity within the range of research topics represented. To portray the nature of this undirected coordination, we have found it instructive to organize the present report around a diagram, in which the various aspects of reality that may or may not be recognized in a particular study or model, are represented by intersecting circles. While such a scheme could easily become a straitjacket if taken too seriously, it may help in seeing the spontaneous interlocking of pieces of research in progress or completed.

Figure 1 provides one place or another for the larger part of the theoretical and empirical work of the last three years. It is based on a four-way classification as follows:

inside A	equilibrium of many persons, firms or sectors
outside A	models with a single decision-maker or sector
inside B	processes extending over time
outside B	one-time choice (or interacting choices)
inside C	optimization models
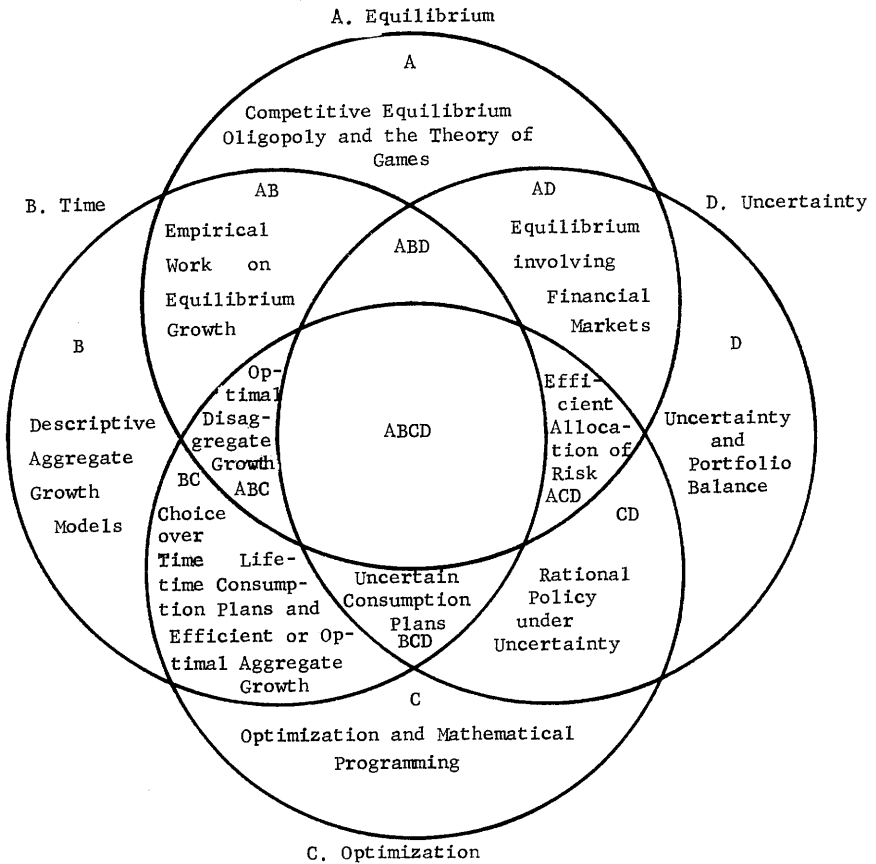
outside C   explanatory, descriptive models (including description in terms of optimization by individuals or firms)

inside D   models recognizing uncertainty

outside D   models assuming complete certainty

Captions in each of the spaces A, AB, ABC, etc., created by circle intersections in Figure 1 refer to the research descriptions that follow under headings preceded by A, AB, etc. Since oppositely placed circles do not intersect
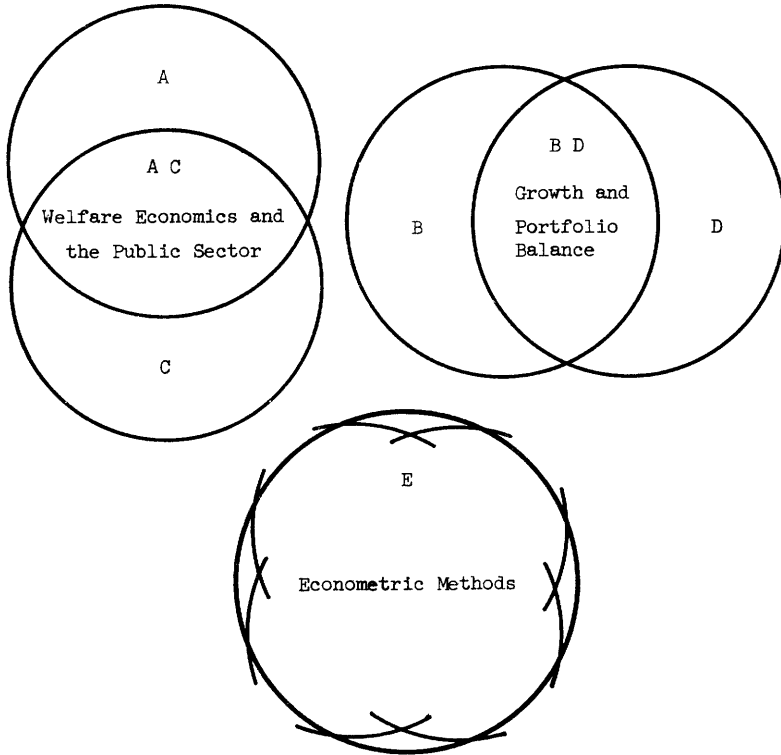
in the diagram at the left of the figure, the categories AC and BD are noted separately on the right. The category E, econometric and other research methods, which in fact penetrates all others, is also displayed separately on the right.

A. *Competitive Equilibrium, Oligopoly and the Theory of Games.*

A general description of an economic system requires both a model of production, and some assumptions concerning the manner in which con-

sumers' demands are generated. While there is an extensive theoretical literature* concerning both sides of the economy, production and demand, the development of algorithms for the numerical solution of economic problems has gone much further for the production side considered by itself, than for the equilibrium analysis in which supply and demand conditions are brought together.

The demand side of the economy is customarily introduced by specifying consumers' demands for commodities as a function of their prices, or, going deeper, by specifying the preferences of the individual consumers that in turn determine the demand functions. In the latter case, a competitive equilibrium may be defined as a price system which equilibrates supply and demand in all markets if each consumer is choosing the most preferred consumption within the constraints of his budget, and if each producer selects a collection of productive techniques which maximizes profit.

In order to demonstrate the existence of equilibrium prices in such a model, it has been customary to make use of what are known in mathematics as "fixed point" theorems. These theorems describe conditions under which a "continuous mapping" of a set of points into itself leaves at least one point unchanged. In the present application, "points" represent alternative commodity price lists.

One mapping, whose fixed points correspond to equilibrium prices, may be constructed by transforming a given list of prices into a new list whose entries have been increased for those commodities in excess demand and decreased for those in excess supply. Equilibrium prices are precisely those which are not changed by the mapping.

An algorithm to compute fixed points of a continuous mapping, and therefore equilibrium prices in a pure exchange model was recently developed by Scarf, (CFDP 216) taking as his point of departure work by C. E. Lemke,** in conjunction with J. T. Howson, on the calculation of a Nash-Cournot equilibrium for a two-person non-zero-sum game. Scarf's algorithm begins by constructing a large but finite collection of price lists, each containing only nonnegative prices whose sum is unity. By distributing these lists (hereafter "points") in more or less even density over the entire price space (see Figure 2 for an illustration in terms of three commodities) one makes sure that at least one price point provides a good approximation to

*Contributions by the Cowles Foundation to this literature are found in Monographs 13 and 17, listed on p. 46.

**Lemke, C. E and J. T. Howson, "Equilibrium Points of Bi-Matrix Games," *SIAM Journal*, Vol. 12, July, 1964.

Lemke, C. E., "Bimatrix Equilibrium Points and Mathematical Programming," *Management Science*, Vol. 11, No. 7, May, 1965.

FIGURE 2   PRIMITIVE TRIPLE IN PRICE SPACE
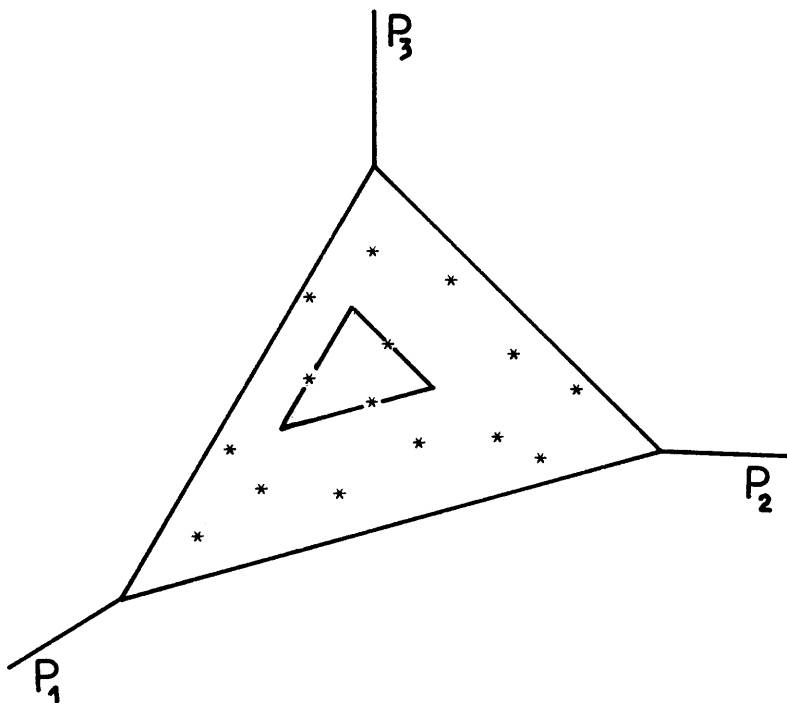
the equilibrium prices. The procedure for finding such a point is similar to (but not identical with) the well-known simplex method of linear programming. Analogous to the "basis" in linear programming is the "primitive set" of as many price points as there are commodities. In Figure 2, a primitive triple permits constructing on it a triangle (broken lines) with sides parallel to the coordinate planes such that no other price point lies inside the triangle. If one removes one point from a primitive set, there is in general one unique replacement for it that will again, together with the remaining points, form a primitive set. The procedure keeps replacing points one-at-a-time *until* a primitive set is obtained such that with each of its points one can associate a *different* commodity, whose demand exceeds the supply if that particular price list were to guide both producers and consumers. The procedure is so designed that this is bound to happen ultimately, and that for any price point then in the "terminal" primitive set

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equilibrium is nearly achieved. Experiments have shown that the algorithm works remarkably rapidly, rarely requiring more than one or two minutes of computation time (on an IBM 7094 machine) for problems involving eight or ten commodities. It is likely that, with improvements in the programming, an extension to fifteen or twenty commodities would offer no serious difficulty.

In Scarf's contribution to the Fisher centennial volume, "On the Computation of Equilibrium Prices," this algorithm is extended to cover the case in which production is described by an arbitrary activity analysis model.

The problems of distribution in a general economic model may also be analyzed by the techniques of n-person game theory, and particularly by that solution concept known as the core. A generalization of Edgeworth's contract curve, the core of an n-person game consists of those production and distribution decisions on which no coalition of n or fewer persons can improve, from its own point of view, using only its own assets and productive technology. It may be demonstrated that any competitive equilibrium is necessarily in the core, and that as the number of consumers becomes larger and larger the core shrinks down to the set of all competitive equilibria.

In "The Core of an n-Person Game," (CFDP 182-R), Scarf stated a general theorem giving sufficient conditions for an n-person game to have a nonempty core. (Many games, including some of those most closely connected with political models, have a degree of instability characterized by an empty core). The theorem, which is applicable to games arising from economic models in which customary convexity and constant-returns-to-scale assumptions are made, may be demonstrated in a constructive manner by means of the same general algorithm used to approximate equilibrium prices, even though the mathematical connection between these two problems is by no means obvious. A variation of this algorithm was developed, in CFDP 211, in order to take advantage of special features which arise if each consumer has a piecewise linear utility function.

The same concern with computational implementation of economic models is present in some of the work of Martin Shubik. Jointly with Richard Levitan of the International Business Machines Corporation, he is engaged in the construction, analysis, and experimental use of a computerized model of an oligopolistic market (CFDP 180, 219, 224, 225, 227). A detailed mathematical investigation of market structure has been given, and the possibility of estimating the parameters of an oligopolistic structure has been investigated. It is expected that this work will in due course be published as a monograph.

In another collaboration of long standing, Shubik and Lloyd Shapley,

of the RAND Corporation, are completing a monograph, "Competition, Welfare, and the Theory of Games," to be published on behalf of RAND in the next few years. Offshoots of this more purely theoretical work that come under the present report are CFDP 204, a general discussion of the application of game theory to the study of economic behavior, and CFP 250, an application of game theoretic solution concepts to the problem of land ownership.

Perhaps the most important insight resulting from the work of Shapley and Shubik is that, under appropriate "classical" assumptions, widely different theories of behavior, whether based on decentralization of allocative decisions, on the power of coalitions, on individual economic power, on efficiency of allocation, or on fair division, all require or imply the operation of a price system, whether that was initially postulated or not. However, when the assumptions are modified to recognize increasing returns to scale and/or external economies, the different solution concepts yield highly different results.

The parallel study of different definitions of "solution" under a variety of assumptions is likely to help in establishing additional links between general equilibrium theory and welfare economics.

James Friedman extended the noncooperative approach to the theory of oligopoly in studies* initiated at the Cowles Foundation and completed while on a year's leave at the University of California at Berkeley. The assumptions of the classical work in this area by Cournot, Stackelberg, and Bowley, have the following implications: a) firms make decisions in any one period so as to maximize their profits in that period and b) the assumption which a firm makes about its rival's behavior is generally incorrect. The natural objection to a) is that firms, realizing they will be in business in future periods, will maximize a discounted stream of future profits over several or many periods. As to assumption b), the objection is that firms will not cling to incorrect assumptions about the behavior of their rivals in the face of evidence to the contrary.

Friedman has dealt with differentiated products in oligopoly models in which: a) each firm seeks to maximize a discounted profit stream, b) each firm behaves according to a "reaction function" which gives the current price decision of the firm as a function of the price decisions of all firms in

*In addition to CFDP 234, see Working Papers of the Center for Research in Management Science at the University of California, Berkeley, No. 188, "Reaction Functions and the Theory of Duopoly," (forthcoming CFP) and No. 213, "A Non-cooperative View of Oligopoly," which is an earlier version of CFDP 234. The latter corresponds to the description above, while the former deals with a model which can be interpreted as a homogeneous product, quantity model or a differentiated products, price model.

the preceding period and c) each firm also knows the reaction functions used by each of its rivals. Under these conditions, it has been proved that noncooperative equilibria, in terms of both prices and reaction functions, exist for a wide class of models.

The nature of the equilibrium is as follows: (a) the equilibrium reaction functions together define a unique fixed point, that is, a unique set of last period's prices which, fed into the equilibrium reaction functions, gives rise to an identical set of this period's prices; and (b), if any one firm were to calculate its profit maximizing reaction function (taking as given the reaction functions of all other firms) and substitute that for the one associated with it in the equilibrium set, the resulting set of reaction functions would still have the same fixed point.

Besides, and prior to, the above theoretical work, Friedman carried out a number of oligopoly experiments. Our preceding Report described experiments which were similar to the Siegel-Fouraker experiments* in complete-information, quantity-adjuster oligopoly. During the period of this report, these experiments were analyzed anew (CFDP 174) in a more refined manner than originally, and a new descriptive model was proposed in which individual behavior was assumed to follow a first order Markov process. It was found that subjects are more cooperative, the more cooperative are their rivals; and that the proportion of symmetric games in which the joint maximum (that is, the maximum possible *sum* of all players' profits) is achieved declines rapidly as the number of players in the games is increased.

Another group of experiments involved the running of a set of complete-information duopoly games, in which subjects were permitted to communicate by means of written messages before each decision. The first of these experiments (CFDP 192) indicated that subjects agree on a joint action approximately 80% of the time and that they agree and honor their agreements 74% of the time, whereas one or both fail to honor their agreement 6% of the time. It is interesting to note that, in games in which agreements are not binding, failure to honor agreements is not frequent.

How frequently are decisions Pareto optimal for the players (that is, allow each player the maximum profit compatible with the profits attained by all other players) ? It was found that agreed Pareto optimal decisions may be expected in asymmetric games some 75% of the time, and in symmetric games 95% of the time. In the absence of agreement, Pareto optimal decisions may be expected approximately 25% of the time.

In asymmetric games the following important solution concepts represent

*Fouraker, Lawrence E. and Sidney Siegel, *Bargaining Behavior,* New York McGraw-Hill Company, 1963; especially Part Three.

distinct points in the Pareto-optimal set: the joint profit maximum, the Nash cooperative game solution, and the point of equal profits. The distribution of Pareto-optimal points obtained in the asymmetric games was found to cluster around a center very close to the Nash solution, except for a subset of one tenth of the observations, which formed an apparently separate distribution centered near the equal split point. While it thus appeared that the Nash solution is attractive to the players, and the equal split point moderately so, more research on this matter seemed called for.

The second set of message experiments (CFDP 207*) was designed so that, in the pay-off matrices, the Nash and joint maximum points were further apart than previously, and the equal split point was moved nearer to both of them. Overwhelmingly the equal split point was chosen in preference to the other two. As a secondary choice, subjects sometimes went to the joint maximum. The Nash solution was almost never chosen. These results indicate that in the circumstances of this experiment the subjects are attracted to equality and, to some extent, by the prospect of jointly earning as much as possible.

Additional experimentation will be needed to determine in what kind of circumstances the different solution concepts come to the fore.

B. *Descriptive Aggregate Growth Models.*

The term "economic growth" has become a short title for the more general study of economic activity as it unfolds over time, for the simple reason that in most applications actual growth is observed, projected, or at least hoped for. But regardless of whether or not positive growth is present or is presupposed, the emphasis is on the allocation of output as between consumption and saving (investment), on the degree and character of technical progress, and on the implications of these variables for the future growth and composition of output.

A descriptive theoretical model of economic growth that has become a basis for empirical work described under AB below was developed by Robert Solow (Massachusetts Institute of Technology), Tobin, Christian von Weizsäcker (University of Heidelberg) and Yaari. In the model of this "4-author paper" (CFP 241), already mentioned in our preceding report, labor is at any time assigned, necessarily and exhaustively, to that plant-and-equipment that confers highest productivity to it, the productivity and the labor-to-capital ratio depending in each case exclusively on the vintage (year

*Forthcoming in Heinz Sauerman, ed., *Beiträge Zur Experimenteller Wirtschafts-forschung*, Volume II.

of original construction) of the capital in question. Unlike previous vintage models of the "putty-clay" variety (examples of which were discussed in our last Research Report), this model permits neither *ex ante* nor *ex post* substitution of the two factors, labor and capital. Yet, because more than one vintage of capital is in use at any time and these different vintages have different (fixed) labor-to-capital ratios, a considerable number of propositions usually associated with models with greater scope for factor substitution are found to hold for this model as well. Given some degree of capital-embodied technological change that is not strictly capital augmenting, diminishing marginal productivity and positive aggregate income shares (at competitive rates of return) occur for each factor. The marginal product of labor declines since additional units of labor will be combined with capital of successively older, and less productive, vintages. Conversely, the marginal product of capital declines since additional units of new capital will draw labor away from successively more recent and productive vintages of already existing capital. This latter property implies that a higher rate of saving (investment) results in an earlier obsolescence of capital of any vintage ("capital quickening"). It is shown that this, in turn, implies a lower market rate of interest. The market rate of interest is defined as that discount rate which equates the present value of future returns (assumed to decline continuously) from a unit of capital to its initial price. It is found to be equal to the social rate of return to saving. Further, if technical change is purely labor-augmenting, there is a "Golden Rule" path of proportional growth on which, given the rate of growth of the labor force, consumption is higher at any point of time than along any other proportional growth path. Along this path, as in other models of Golden Rule growth, discussed under BC below, the saving rate is equal to the share of capital in gross product and the rate of interest is equal to the growth rate. The long-run behavior of the economy, given a constant saving rate not necessarily equal to the "Golden Rule" rate, is also explored.

AB. *Empirical Work on Equilibrium Growth*

Combining a theory of production (such as that in the 4-author paper) and a theory of effective demand into a cohesive empirical model would be a key step toward providing answers to the numerous policy questions that can be asked about U.S. economic growth. The previous Research Report listed some of these questions which, it is hoped, can be answered by a joint MIT-Yale Study of Future U.S. Economic Growth (FUSEG) once a complete model is constructed. Members of the Cowles Foundation staff who have devoted time to the project during the last 3 years include: Attiyeh,

Bodkin, Drandakis, E. and C. Phelps, Mieszkowski, Nerlove, and Tobin.

Estimation of a satisfactory production model is at the heart of the project. Members of the Yale group working on the FUSEG project have explored various aspects of production models and work has proceeded on the empirical estimation of a few of these. In Bodkin's "Nonlinear Estimation of Aggregate Production Functions," (CFP 252), written jointly with Lawrence R. Klein of the University of Pennsylvania, a variety of production functions are estimated for the U.S. private non-agricultural sector, over the period 1909–1949. These include a general constant-elasticity-of-substitution model and a simple Cobb-Douglas model. The paper concludes that the elasticity of substitution, for an aggregate production function, is probably intermediate between the values of zero (the fixed-proportions special case) and unity (the Cobb-Douglas case). The conclusion of an earlier report on this study (CFDP 157), that the American economy appears to operate in the range of increasing returns to scale, continues to hold.

Edmund and Charlotte Phelps estimated a "vintage" production model of aggregate private non-farm output in the U.S. economy (CFP 253) using the concept of the factor-price-frontier, introduced by Paul Samuelson of the Massachusetts Institute of Technology. The theoretical aspects of the general model—which admits embodied and disembodied technical progress and both *ex ante* and *ex post* substitution of capital and labor—were developed by Solow* and by Phelps (CFP 188). From a growth equation that relates the increase of output to the rate of investment, to the increase of employment and to time as a proxy for technical progress, it is possible to infer the time series of the "marginal productivity of investment." This series, in combination with the real wage rate (as a proxy for the marginal productivity of labor), permits estimation of the aggregate production function without use of capital stock data. The results obtained, though not highly reliable, agree, for the most part, rather closely with some previous studies of such a "vintage" model by Berglas and by Intriligator,** who did use capital stock data. The results revise downward earlier estimates of the social rate of return to investment, and display fluctuations in the rate of return.

The "4-author paper" model described above provides an interesting

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*"Investment and Technical Progress," in Arrow, Karlin and Suppes, *Mathematical Methods in the Social Sciences*, Stanford University Press, Stanford, 1960.

**Berglas, E., "Investment and Technological Change," *Journal of Political Economy*, April, 1965.

Intriligator, M., "Embodied Technological Change and Productivity in the United States, 1929–58," *Review of Economics and Statistics*, February, 1965.

alternative model for empirical estimation. On the one hand, the stronger *a priori* assumptions about the nature of factor substitution possibilities somewhat simplify some estimation problems and offer the hope of greater reliability. On the other hand, the model conforms to the casual observation that there is considerable scope for shifting factor proportions in the long run, as a result of technological change, but quite limited scope in the short run. The potential usefulness of this model in empirical estimation has been demonstrated by Attiyeh in "Estimation of a Fixed Coefficients Model of Production" (CFDP 210). The estimated production coefficients imply that technical change is approximately labor-augmenting, that about half of that technical change is embodied in new equipment; and that over the period of observation substantial capital "quickening" occurred. The factor price implications of the estimated model appear to be consistent with actual market prices. Thus, both the factor price implications and the findings on the structure of technology seem quite plausible. Nevertheless, the model still presents considerable problems of estimation; efforts have been under way, and are continuing, to find a practical computational method for routine use of the model, including its application at a partially disaggregated level.

On the demand side of the FUSEG model, work at Yale has been focused particularly on components of total spending other than business capital outlays. Bodkin's work on consumption-personal-saving relationships* was presented at the Econometric Society meetings of December 1966. A key aspect of these relations is the use of the same explanatory variables in all the regressions; hence the coefficients on disposable income add up to unity across the category relationships, while the coefficients of all other explanatory variables add up to zero.** This means not only that the identity between actual measured income and the sum of all expenditures and personal saving is preserved, but also that this identity will automatically hold for the values predicted by the estimated relationships. The explanatory variables are annual disposable income (real, per capita), household net worth (real, per capita), stock of consumer durables (real, per capita), and two relative price variables. One substantive conclusion of this analysis is that the coefficient measuring the effect of household wealth on annual consumption is estimated to be around .03 or .05, depending upon how one classifies the current year's expenditures on durable goods.

*"The Consumption Sector of the Yale-MIT Model of Future U.S. Economic Growth."
**This technique is closely related to one presented by R. Stone in "Models for Demand Projections" in *Essays on Econometrics and Planning*, Pergamon Press, Oxford, 1965.

Bodkin has also studied the determinants of investment expenditures on new residential construction, in the context of a longer term outlook which abstracts from the starts-inventory-completions mechanism. The tentative conclusions of this study (which is still in a preliminary stage) conform to intuition. The study finds outlays on residential construction to be positively influenced by the growth of disposable income and by a rise in the ratio of the rental cost of housing services to a price index for residential construction, and to be negatively related to the growth of the stock of housing itself. Financial variables, such as the ratio of loan value to housing value on new mortgages and an average yield on new mortgage loans (particularly the former), also appear to be pertinent explanatory variables. These conclusions seem to carry over to a possibly more desirable formulation in which construction outlays, disposable income and the housing stock are put on a per family basis.

The intended plan of the over-all FUSEG model calls for disaggregating the production side into about 13 major producing sectors (agriculture, manufacturing of durable and nondurable intermediate and final products, construction, transportation, utilities, and several service and government sectors). Since these production categories do not correspond with the conventional categories of expenditures, one aspect of linking the production and demand sides of the model is the construction of a set of "input-output" transformations that will distribute demand among production sectors. This work is being done at MIT. Factors of production must also be allocated among production sectors. Demand for factors (capital and labor) is implied by the production relations. The required structure of differentials among wages, and among rates of return to capital, in various producing sectors must be determined. Preliminary work on these projects has been done at Yale by Attiyeh, John Koehler, Daniel Radner and Dennis Smallwood.

The distribution of income between the business and household sectors of the economy, another necessary link, can be developed in a fairly straightforward fashion from the returns to factors. Tax relationships, determining the distribution between the public and private sectors, were being developed by Mieszkowski toward the end of the period of this Report. Mieszkowski is also working on other aspects of the behavior of the public sector including the contributions of some types of Government expenditures to the production process.

Alternative tableaux simulating the economy at points in the intermediate future under various assumptions are the yet unrealized objective of the FUSEG model. It is expected that they will be run under the general as-

sumption that the stabilization policies maintain full employment; neither the supply nor demand sides of the model are designed to describe the dynamics of the under- or over-employment. Nevertheless, simulations based on this assumption should point to the requirements for fiscal and monetary policies that would preserve full employment while achieving a given growth target.
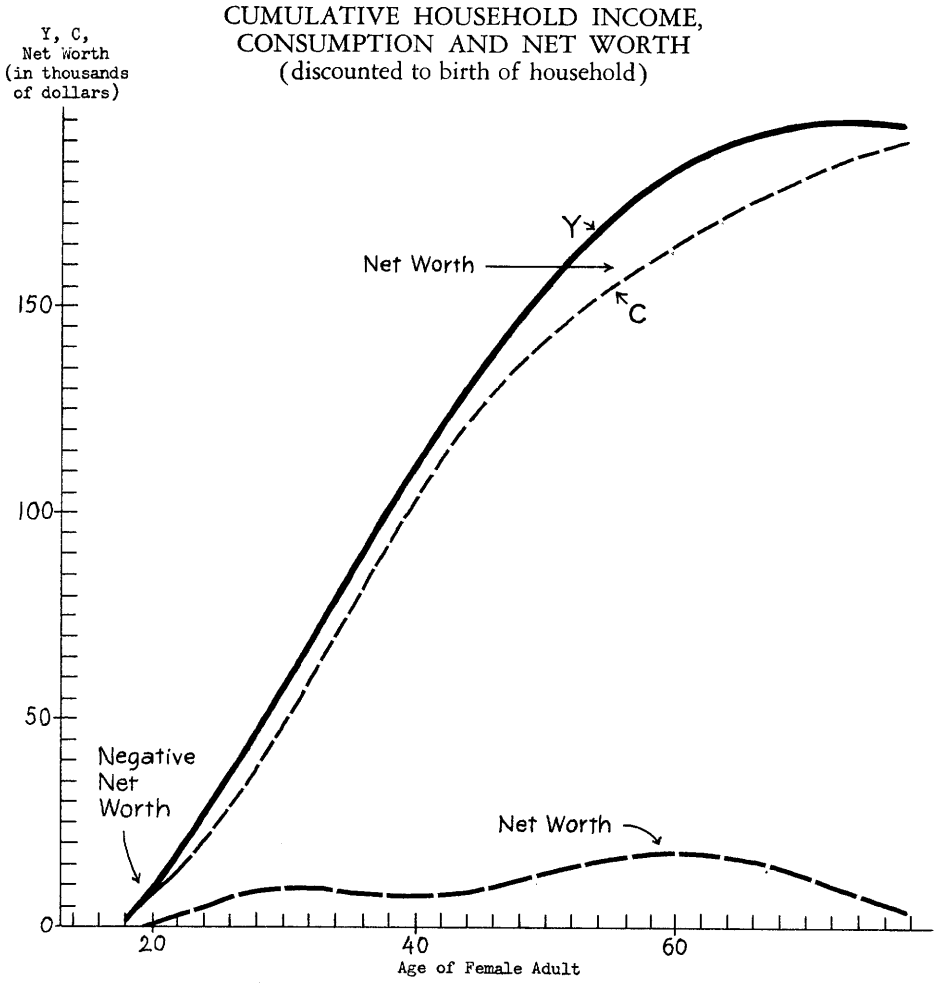
In a somewhat different vein, Shubik has worked (at some stages, in collaboration with Friedman) on the construction of a socio-economic simulation of a Latin American economy. The model constructed in CFDP 203 distinguishes, in addition to the usual categories of a dynamic GNP model, four social classes, and makes provision for social mobility. This enables at least rudimentary consideration of the problems of income distribution among social classes instead of the usual distribution among factor shares. Even a crude simulation which enables the modeler to attempt the inclusion of such sociological (and political) variables seems likely to be fruitful in studying the growth of developing economies. The model is, as yet, illustrative and has not been estimated for a specific country. A basic argument for this approach, however, is that simulation and econometric analysis of a socio-economic process can be developed and used jointly. Thus, the construction of a relatively crude simulation may be useful in organizing data and justifying the need for the gathering of statistics. It can be looked upon as a means of promoting discourse among economists, data gatherers, and administrators.

The proportions in which a society divides its income between current consumption and saving play an important role both in short-run fluctuations of business activity and in the long-run growth of the economy. For this reason the search for reliable regularities in consumption-saving decisions has long been a major focus of economic inquiry, both theoretical and empirical. Tobin prepared a survey of the field for the forthcoming *Encyclopedia of the Social Sciences*, (The Consumption Function).

One plausible theory of saving is the "life cycle" theory. In its most general form the theory is due to Irving Fisher. In various papers discussed in section BC below, this theory has been extended and related to the theory of growth. According to a life cycle model, a consumer unit—an individual or a household—saves or dissaves at different ages as necessary to keep its consumption stable in the face of fluctuations of income. The chief example is saving for retirement; another illustration is borrowing, in anticipation of future income, by young families, to finance graduate education or early years of professional practice. The collective result of this kind of behavior —if anticipatory savings outweighs anticipatory borrowing—will be net

FIGURE 3

Y, C,
Net Worth
(in thousands
of dollars)

CUMULATIVE HOUSEHOLD INCOME, CONSUMPTION AND NET WORTH
(discounted to birth of household)

Age of Female Adult

saving in a growing economy, even though no single consumer unit saves any of its income, on balance over its full lifetime. In "Life Cycle Saving and Balanced Growth," forthcoming in the Fisher centennial volume, Tobin has calculated how important this source of saving might be in an economy resembling, in its rate of growth and age-income profile, that of the United States.

Figure 3, reproduced from Tobin's article, illustrates part of the calculation. The Y curve shows for each age of wife the cumulative income of a representative American household up to that age. The C curve shows cumulative consumption; it is drawn on the assumption that the household exhausts its lifetime income and, with allowances for changes in family size, consumes evenly over its lifetime. The difference of Y and C gives the net worth of the household, and the change in this difference from one age to the next is net saving. For the economy as a whole, the various ages must be weighted in proportion to their numbers in the population. It appears from such aggregation that life cycle saving, without any new saving for future generations, can plausibly account for total household wealth in the United States.

The permanent income hypothesis is one rigid version of the life cycle hypothesis of saving. In CFP 233, Bodkin and Roger C. Bird (Lafayette College) examine the evidence for a strict interpretation of the permanent income hypothesis which calls for the marginal propensity to consume transitory income to be zero. They used data on income and expenditures of a subsample of veterans who had received a one-time extra National Service Life Insurance dividend in 1950 and who fell within the 1950 Bureau of Labor Statistics survey of households. Bodkin had used this data in an earlier paper;* in CFP 233, the authors re-worked his analysis, seeking, in particular, a better measure of, or proxies for, permanent income. On the basis of this analysis, the strict permanent income hypothesis could not be rejected. The evidence seemed to lend itself somewhat better, however, to a weaker version of the hypothesis requiring an appreciably smaller propensity to consume transitory income than permanent income.

C. *Optimization and Mathematical Programming.*

Theoretical work at the Cowles Foundation on concepts and techniques of optimization for an individual economic unit or for the economy as a whole has been described in earlier reports. Work in the present report

*"Windfall Income and Consumption," *American Economic Review*, September, 1959.

period involves mainly optimization over time, or under uncertainty, or both. One piece of work concerning optimization technique pure and simple should be reported here.

Whinston, in collaboration with C. van de Panne of the University of Groningen, Netherlands, completed a paper on Houthakker's quadratic programming algorithm (CFP 231). The paper reformulates Houthakker's capacity method for quadratic programming in the framework of the simplex and dual methods for quadratic programming, thereby reducing the conceptual and computational complexities of the method. It is shown that the method is applicable for all convex quadratic programming problems, including the case of a semi-definite matrix of the quadratic form, and that of constraints in equality form. In the linear programming case, the method reduces to a parametric version of the dual method.

AC. *Welfare Economics and the Public Sector*

Under a number of familiar assumptions, market equilibrium in a perfectly competitive economy in which public goods are absent or unimportant is associated with a somewhat limited optimality property usually referred to as Pareto optimality. In an economy where a government raises revenue in order to provide certain goods or services that must be collectively consumed, however, the market mechanism alone does not determine either resource allocation or final income distribution. Optimum equilibrium patterns of resource allocation in such an economy are more difficult to define and describe.

One of the best established lines of inquiry in this general field is concerned with the efficiency of alternative tax and transfer schemes (assuming the total amount of revenue is given). A closely related line of inquiry is concerned with the incidence of such schemes. Mieszkowski worked on both these topics, continuing his work during the year 1964–65 as a post-doctoral fellow at the University of Chicago. In one paper (refer to [A] in author's list) he compared the relative excess burden associated with tariffs and other tax systems and concluded that except under very special conditions tariffs are *relatively* inefficient as a means of raising revenue or stimulating the production of a particular sector of the economy. In another paper (refer to [B] in author's list), using the well known two-factor two-commodity model common in trade and growth theories, he derived conditions under which different tax systems are equivalent in their distributive effects, and conditions under which a substitution of one type of tax finance for another results in an unambiguous increase or decrease of after-tax income for a

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particular group. In closely related work on the incidence of the property tax, he argued that, contrary to conventional wisdom which maintains that property taxes are shifted to consumers similarly to excise taxes, property taxes are much more like profits taxes.

In collaboration with John Cragg and A. C. Harberger, of the University of Chicago, Mieszkowski developed a critique (refer to [C] in author's list) of the Krzyaniak-Musgrave book* on the incidence of the corporate tax in manufacturing. It is shown that these regression results are biased upwards because of misspecification of the regression model. Also, it is argued that, if capital markets are perfect, the *after-tax* rates of return in the corporate and non-corporate sectors will be equal and imposition of, or increase in, corporate tax rates will increase before-tax profit rates in corporate manufacturing. A model very similar to this was applied by Mieszkowski and Harry G. Johnson of the University of Chicago to an analysis of the effects of trade unions. They found that, under a wide range of assumptions on various parameters, the gains of union labor are made primarily at the expense of nonunion labor and not at the expense of owners of capital.

Recently, in discussions of economic integration and of the harmonization of tax systems between countries, there has been some theorizing as to whether indirect taxes should be based on the destination principle, (i.e., exempt exports and tax imports) or on the origin principle. Mieszkowski showed in a note that, contrary to the recommendations of the Newmark Committee and the findings of other research in this area, there is no *a priori* basis for preferring the origin principle to the destination principle on efficiency grounds.

Tobin and Mieszkowski, in collaboration with Joseph Pechman, Irving Fisher Professor of Economics at Yale in 1965–66, on leave from the Brookings Institution, have been working on aspects of the negative income tax. They have been concerned with the feasibility and incidence of such an income maintenance scheme, and with its implications for economic efficiency compared with other types of income transfers to the poor.

On the public goods side, Brainard (while on leave at the Brookings Institution) and F. Trenery Dolbear (Carnegie Institute of Technology) have considered the problem of externalities that arises when local governments make independent decisions. They took exception, in a comment (see [A] in the author's list), to the proposition, put forth by Williams,**

*"The Shifting of the Corporate Income Tax," *An Empirical Study of Its Short-Run Effect Upon the Rate of Return*, Baltimore, Johns Hopkins Press, 1963.

**Williams, Alan, "The Optimal Provision of Public Goods in a System of Local Government," *Journal of Political Economy*; February, 1966.

that it is impossible to tell *a priori*, even in the highly simplified model Williams presents, whether independent decisions of local governments about the supply of public goods will lead to aggregate under- or over-supply compared with a situation analogous to centralized government decision making.

Fully general consideration of optimality in an economy where resources are used in the production of public goods—i.e., goods that are collectively consumed—requires consideration of public goods and taxes jointly. Since provision of public goods and the corresponding imposition of taxes requires "collective" decision making, some political decision rules must be incorporated into the analysis. In a doctoral dissertation* at Yale, written under the supervision of Scarf, Tobin, and William Fellner, Duncan K. Foley developed a characterization of the set (or surface) of Pareto optimal combinations of public and private goods involving a political decision rule calling for unanimous consent in shaping the public sector program. He also demonstrated that it would be possible to determine a Pareto-optimal combination of public and private goods so that each individual's after-tax income available for purchases in the private sector would be proportional to his income before taxes. The tax rate is determined implicitly by the interaction of individual preferences for public versus private consumption and by the costs of production in the public sector.

Shubik has developed a taxonomy of public goods (CFDP 208) based on questions and considerations pertinent to the formulation of models involving such goods. This taxonomy includes such criteria as whether there is a possible, and legal, private substitute for the public good in question; for example, private schools are a substitute for public education, but private action may be ineffective in controlling contagious diseases, and it is illegal to maintain a private army.

The interaction of economic and political questions can also be approached from the direction of political science. While a guest at the Cowles Foundation, Gerald Kramer of the Department of Political Science of the University of Rochester examined the relationship between short-term economic fluctuations and U.S. Congressional election outcomes. His modelling of the election process, which was developed from a "satisficing" hypothesis, focused on the vote for or against the incumbent party and permitted discrimination among several alternative hypotheses about the nature of long-term changes in political alignments of the U.S. electorate. He found that fluctuations in real personal income have a considerable effect on the vote for the incumbent party—under any of the alternative versions

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* "Resource Allocation and the Public Sector," *Yale Economic Essays*, Spring, 1967.

of the hypothesis tested—while little independent, additional effect was detected for changes in unemployment and in the consumer price level.

BC. *Choice Over Time, Lifetime Consumption Plans, and Efficient or Optimal Aggregate Growth.*

In problems of optimization over time, it is often assumed that the consumer, firm or planner seeks to maximize the discounted sum of future utilities. In CFDP 206, Koopmans has given an axiomatic discussion of this formulation of the objective of economic choice. The axioms are concerned with a preference ordering of consumption programs (infinite sequences of future consumption bundles), and express continuity, sensitivity, and stationarity of the preference ordering, its monotonicity for an infinite sequence of program changes, and (regrettably) noncomplementarity of consumption bundles associated with different future periods.

Our preceding report mentioned several studies by Yaari of optimal lifetime consumption planning (now published as CFP's 220, 223 and 230). Watkins pursued several distinct further studies of this topic that extend the model in various directions.

The simplest and possibly most interesting extension is in terms of an additional constraint on the allocation of consumption over time. The standard model for decision-making under certainty includes a restriction that an individual cannot be in debt at the end of the planning period. This constraint can be generalized to a restriction that an individual's net worth cannot at any time drop below an institutionally imposed bound, which may vary with the age of the individual, and be at times negative. The analysis of this model turns out to be simple and suitable for diagrammatic presentation. The optimal consumption program can have time segments of two types: (1) segments where the net assets are above the lower bound imposed by institutions: hence the analysis of the standard model applies, and (2) segments where the net assets are for some time exactly equal to the bound; hence consumption is determined by current income and changes in the "debt limit." The analysis gives a method for determining the division of the life span into these various segments. The results can be used to construct examples illustrating the effect of various factors (such as windfalls) on consumption, and to clarify the concept of normal income. The results are also useful in analyzing more general cases of credit restrictions such as interest rate differentials between borrowing and lending.

Further studies by Watkins concern (a) a formulation of the objective of consumers' choice in terms of states or conditions produced by consump-

tion of goods and services rather than the acts of consumption or purchase themselves, and (b) the case where preferences change over time.

We turn now from models referring to the individual consumer's lifetime allocation to highly simplified models of optimal aggregate growth for an entire economy. Our preceding report described work by Phelps on the "golden rule of capital accumulation," which indicates how to sustain a proportional growth path on which consumption is uniformly higher than on any other feasible proportional growth path. Phelps' book, *Golden Rules of Economic Growth*, [Norton & Co., New York, 1966] expounds, generalizes, applies and extends the concept of the Golden Rule path in a number of simple economic models. The meaning and conditions for the existence of the Golden Rule path are set forth. Certain analogues and generalizations of the Golden Rule path are presented. Finally the Golden Rule concept is extended to kinds of investment other than tangible capital formation. For example, a Golden Rule of Research is derived from a model of technological progress and production. On the assumptions of this model, it is shown that, among all proportional growth paths, the path yielding uniformly highest consumption—the Golden Rule path—is characterized by equality between the rate of return to investment in technology and the rate of growth; if capital were unproductive in research and the labor force were homogeneous, exactly half the labor force would do research on this Golden Rule path!

A chapter in this book builds on earlier work by Phelps and Richard Nelson on one particular role of education in economic growth to which attention had been drawn earlier by Nelson. The principal thesis of their joint paper, "Investment in Humans, Technological Diffusion and Economic Growth," (CFP 236) is that education is of special value to those actors in the economy, production managers largely, whose main function is the evaluation and introduction of new techniques of production. Hence the diffusion of technical progress is enhanced by the education and consequent technological sophistication of those involved in choosing the means of production. As a consequence, the value of that type of education and the payoff to society's investing in that type of human capital are likely to be greater the faster is the rate of technical progress.

Drandakis and Phelps have examined the effects of factor augmenting technical progress in response to economic incentives. An invention possibility frontier is introduced, following Kennedy,* which indicates the maximum possible rate of labor augmentation corresponding to any given rate

*Kennedy, C., "Induced Innovation and the Theory of Distributive Shares," *Economic Journal*, 1964.

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of capital augmentation. This frontier is stationary over time, and the "firms" are assumed to maximize the current rate of technical progress. Long-run equilibrium is possible and will gradually be reached if the saving-income ratio is constant or declining over time. Only in the case of purely labor augmenting progress is there a golden-age equilibrium. However, distributive shares are constant for both types of progress, and their magnitudes depend only on the shape of the invention frontier.

In an economy with constant technology and exponential population growth the Golden Rule path also serves as a magnet that, in the long run, attracts to its proximity all "optimal" paths, however different in initial capital stock. Here optimal paths are those that maximize an undiscounted sum of future per capita utilities. Work by Koopmans (CFP 238) and Cass,* described in our earlier report, covers this case, as well as analogous cases where optimality is defined in terms of a discounted sum of per capita utilities. Koopmans has prepared a diagrammatic exposition (CFDP 228) presenting the entire range of cases involved. This is to appear in the Fisher centennial volume. It includes the paradoxical case of a discounted sum of individual (*not* per capita) utilities, in which no optimal path exists whenever the discount rate applied to future utilities is less than the growth rate of the population. The reason is that under these circumstances a long-continued buildup of the capital-labor ratio for the mere purpose of some future splurge of capital consumption is scored the higher by the optimality criterion, the more distant the future timing of the splurge. Koopmans also wrote a survey and evaluation (CFDP 212) of recent literature on optimal aggregate growth models that assume certainty about future preferences and technology.

Beals and Koopmans have extended this work to the case of a more general maximand implying a discount rate that can vary with the prospective consumption path, while maintaining the assumption of a constant technology. This paper (CFDP 229) was presented at the Symposium on Mathematical Programming, held in Princeton, New Jersey, August 14-18, 1967.

All of this literature presupposes an indefinite planning period. While from a theoretical viewpoint this assumption is required for consistent treatment of all future generations, from a practical viewpoint the assumption of a finite planning period with terminal capital targets seems more relevant. Cass (CFDP 178) has investigated the implications of maximizing the sum of discounted utilities of per capita consumption over a finite period subject

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*"Optimum Growth in an Aggregative Model of Capital Accumulation," *Review of Economic Studies*, July 1965, pp. 233–240.

to a terminal capital constraint, again within the context of the single-good, constant-technology model of capital accumulation. His principal result is to demonstrate a "turnpike" property of the optimum path of capital (and therefore of that of consumption) : For long planning periods the optimal capital path is mostly near that level at which the net marginal productivity of capital equals the rate of growth of the population plus the rate of discount of future utilities (the rate of time preference). In particular, the latter level of capital again traces the Golden Rule path of capital if that rate of discount is zero.

Finally, the Golden Rule path has a central role in the study of efficient growth paths, that is, paths allowing as high a consumption level at any time in the future as is compatible with consumption levels attained by the path at all other times, given the anticipated technology, and the initial capital stock. Continuing the line of investigation begun by Phelps (CFP 232), Cass and Yaari have worked and will continue to work on deriving a useful and yet complete characterization of all the efficient paths of capital in the aggregate growth model described above. Approaching the problem first from the viewpoint of the marginal productivities or real rates of interest associated with each feasible path of capital, they have shown, for example, that a sufficient (but not necessary) condition for efficiency of a feasible path is that the present value of a unit of future capital (or consumption) remains finite as the date of valuation becomes infinite (appendix to [B] in author's list).

Interest in the study of efficient paths has been heightened by recent results demonstrating that competitively determined growth paths of capital need not be efficient. This seems to contradict the central theorem of modern welfare economics that says that, under a fairly wide range of circumstances, competitive allocation of resources will be Pareto optimal. The applicability of this theorem to allocation over time was brought into question by Samuelson,* who showed that competitive equilibrium over an indefinite future in a world permitting trade but no production need not be Pareto optimal; later Peter Diamond,** now of the Massachusetts Institute of Technology, showed that with the addition of production such an equilibrium may even be inefficient. Cass and Yaari have pursued this problem further, both in the very simple context where each individual lives only two periods and production consists only of carrying inventories from one

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*"An Exact Consumption Loan Model of Interest With or Without the Social Contrivance of Money," *Journal of Political Economy*, 1958.
**"National Debt in a Neoclassical Growth Model," *American Economic Review*, December, 1965.

period to the next (CFP 240), and in the more complicated context where individuals live many periods and production consists of using capital and labor to produce output (refer to [B] in author's list of D. Cass). The principal result of both of these studies is the conclusion that nonmarket institutions (for example, a central bank that issues fiat money) may be required to provide enough intermediation between generations to insure that the allocation of resources will be Pareto optimal.

The role of "money," in the sense of assets whose value depends on social agreement or governmental cachet, in providing such intermediation has been explored in another paper by Cass and Yaari (CFDP 215). There it is shown that fiat money (that is, money valued only because of confidence that it will be valued by others in the future) may or may not suffice, while commodity money (valued also for its own sake) will always suffice to insure that competitive allocation of resources will be Pareto optimal. Along somewhat different lines, Drandakis (CFP 242) has also investigated intertemporal general equilibrium in a monetary economy.

ABC. *Optimal Disaggregate Growth.*

Work in this area, described in the previous report, reached publication in the period of the present report, by Drandakis (CFP 244) and Koopmans (CFP 217).

D. *Uncertainty and Portfolio Balance.*

The microeconomic foundations underlying most branches of received economics are supplied by some assumption of optimizing behavior—such as maximization of utility by consumers or of profits by firms—under conditions of perfect certainty, perfect markets, and no transactions costs or other frictions. During the last twenty years, theoretical tools have been developed for dealing directly with the problem of defining optimal behavior in situations involving market imperfections, transactions costs and other "frictions," and uncertainties about future prospects. The tools developed have proved to have a wide range of fruitful applications; developments in monetary theory are a major, but by no means the only, example.

One such theoretical development is the general analysis of decision-making under uncertainty. Some of the earliest work in this field was cast in terms of analyses of gambling. One apparent anomaly that puzzled analysts was the observed phenomenon of the same individual both gambling and buying insurance — i.e., on the one hand, preferring uncertainty over an actuarially equivalent, certain state while, on the other hand, pre-

ferring certainty to an actuarially equivalent gamble. In CFP 227, Yaari argues that the simultaneous actions of gambling and buying insurance need not be explained, as they were by Friedman and Savage,* by a "dent" (that is, a non-convexity) in the decision maker's utility function. An alternative explanation could lie in a divergence between the decision maker's subjective probabilities and the observer's "objective" probabilities. Preliminary experiments show that in some cases this is, indeed, true.

Diversification of risk is another aspect of the analysis of decision making under uncertainty. This analysis has made it possible to give precise expression to the common sense observation that distaste for risk leads a rational decision maker "not to put all his eggs in one basket." This theoretical development has provided one important element in an approach to monetary theory, with which some members of the Cowles Foundation staff have been identified, since it provides a reason for rational investors to diversify their portfolios while simultaneously holding assets with sometimes widely differing expected yields.

In terms of this approach, monetary theory, broadly conceived, is simply the theory of portfolio management by economic units: households, businesses, financial institutions, and governments. It seeks to explain holdings by economic units of stocks of assets and debts (including money proper) and the values and yields of these claims; its accounting framework is the balance sheet. It can be distinguished from branches of economic theory which take the income statement as their accounting framework and flows of income, saving, expenditure, and production as their subject matter.

Like other branches of economic theory, monetary theory has both a microeconomic and a macroeconomic side. Monetary microeconomics concerns the balance sheets or portfolio choices of individual units—households, businesses, or financial institutions. The choices are constrained by the wealth of the unit and by the opportunities available to a person to buy and sell assets and to incur or retire debt. Within these constraints, the choices are determined by the objectives, expectations, and uncertainties of the unit. Monetary macroeconomics concerns the general equilibrium of the capital accounts in the economy as a whole, the way in which asset prices and yields adjust to equate the demands to the supplies of the various assets and debts.

This approach provides the broad conception of "monetary" economics underlying three new Cowles Foundation Monographs edited by Hester and Tobin and published in September 1967. These monographs bring

*M. Friedman and L. J. Savage, "Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, August, 1948.
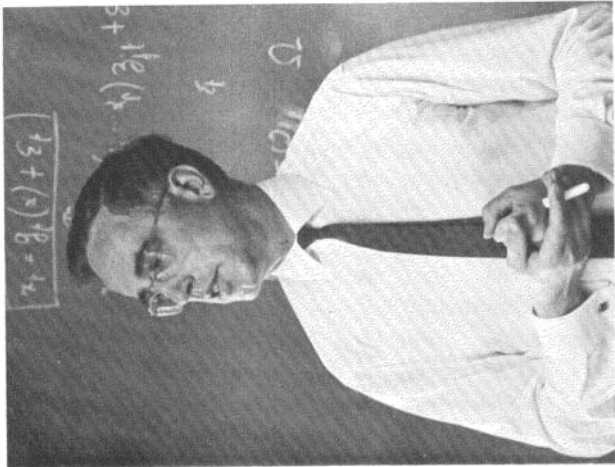
Staff Meeting Addressed by Edmond C. Malinvaud

Staff Meeting Addressed by Michael C. Lovell

together nineteen essays on theoretical and empirical monetary economics written by staff members of the Cowles Foundation (some appeared earlier as Cowles papers) and by recent recipients of the Yale Ph.D. A table of contents for the three volumes is given on p. 47–48.

The seven essays in Monograph 19, *Risk Aversion and Portfolio Choice*, are partly theoretical and partly empirical. They are concerned, on the one hand, with the *attitudes* of investors toward risk and average return and, on the other with the *opportunities* which the market and the tax laws afford investors for purchasing less risk at the expense of expected return.

Monograph 20, *Studies of Portfolio Behavior*, is institutionally oriented. The six essays draw on the theoretical developments mentioned above and seek to apply them to the particular circumstances and objectives of various kinds of economic units: households, nonfinancial corporations, banks, and life insurance companies. The monograph is intended to demonstrate the contribution of these analytical tools to the interpretation of the statistical data available on balance sheets and capital accounts.

Monograph 21, *Financial Markets and Economic Activity*, is concerned with the conditions of equilibrium in economy-wide financial markets. Assuming the microeconomic principles previously discussed guide the behavior of individual economic units, including financial intermediaries, interest rates and other yields must adjust to create equilibrium in various financial markets simultaneously. The theoretical studies in Monograph 21 apply this framework to investigate the consequences of the existence of various institutions and regulations for the effectiveness of monetary control. In addition some empirical findings on the structure of interest rates by maturity and by risk category are reported.

Tobin also has been revising and preparing for publication his book on monetary theory and policy, which is a basic exposition of the approach underlying the Monographs. A paper on the theory of portfolio selection, based on the book, was published by Tobin in the collection of papers discussed at the Royaumont Conference of the International Economic Association.*

Other work subsequent to that in the Monographs has focused on questions related to the portfolio choices of households and firms. Empirical analysis of the portfolios of microeconomic units who are "price takers" requires knowledge of the net returns available to these units on different kinds of assets, and the costs to them of raising funds from alternative sources. Hester, in collaboration with John F. Zoellner of the Federal Reserve Bank of Kansas City, continued research on statistical cost accounting

*Hahn and Brechling, eds., *The Theory of Interest Rates*, Macmillan and Co., 1965.

for commercial banks (CFP 249). Net current operating income, profits before taxes, and profits after taxes were regressed on different bank assets and liabilities for two samples of banks, over a number of years. The regressions yielded plausible estimates of net rates of return which banks earned from the various components of their portfolios. The deposit mix of banks proved to be especially important in explaining variations in after-tax profits of banks in this study. Furthermore, by comparing estimated rates of return obtained from different income measures, Zoellner and Hester were able to report how default losses and taxes affected rates of return earned on different assets. The relation between bank size and the net income measures, given a bank's asset composition, was also studied. Large banks were found to have a higher ratio of net income to total assets for both before-tax income measures. After deducting corporate income taxes, however, no such relation was found. Hester therefore suggests that "economies of scale" in banking may be observed primarily because tax laws favor high-cost small units.

Subsequent attempts of Hester's to apply this method to large cross-section samples of mutual savings banks, savings and loan associations and non-financial corporations were less successful. One explanation may be that individual balance sheet categories contain a large number of heterogeneous assets or liabilities whose mix differs markedly among firms.

Several members of the staff have conducted research dealing with rates of return on corporate shares. Tobin is investigating to what extent differences among stocks in earnings/price ratios or dividend yield can be explained by dividend pay-out policy, trends in earnings, cyclical sensitivity of earnings, volatility of earnings, and the burden of other debt. As these influences are estimated separately for a sequence of cross-sections, changes over time in market evaluations of different stock characteristics can be inferred. One objective is to obtain better measures of the effective cost of capital to corporations of different kinds. Hester and Nerlove have explored the characteristics of firms' income and balance sheet histories which are associated with high rates of return on stock traded on the New York Stock Exchange. For this purpose, return is defined so as to include capital gains. Preliminary results suggest that investors' return on traded equities is most directly correlated to growth of sales. Both of these studies have used the "Compustat" record of stock prices and related data for nearly 900 industrial companies, for a period of 20 years; a separate tape covers public utilities. The Compustat tapes of data have been compiled, and made available to Yale, by the Standard Statistics Corporation.

The portfolio behavior of financial institutions depends on many variables such as their corporate form, their size, relevant regulations and the

characteristics of their liabilities, as well as the net returns achievable from available assets. Some of these variables take on different values both over a cross section and through time, while others change only through time. In addition, a single firm's portfolio adjustment to an exogenous "shock" is not likely to occur instantaneously, and the interaction of firms throughout the financial system will further delay the achievement of a new equilibrium. Consequently aggregation problems may obscure the pattern of adjustment of the single firm if time series data are analyzed alone. For these reasons, Hester and Pierce have used large samples of cross-section, time-series data for analyses of the behavior of financial institutions. The aim of these analyses is to provide information on the behavior of individual institutions and possibly to identify institutional changes that might increase the efficiency of financial intermediation.

Hester studied savings and loan associations in a consulting capacity with the Federal Home Loan Bank Board (CFDP 201,* refer also to [C] in author's list). He explored, in particular, whether differences in associations' corporate form of ownership result in different patterns of behavior. Using an analysis of variance model and eliminating the effects of interstate variations, he found that, during the 1961–64 period, stock chartered associations grew approximately twice as fast as their mutual counterparts. Stock associations also had considerably higher foreclosure rates, larger stocks of reacquired real estate, higher advertising and labor expenses in relation to assets, and lower average account size than mutual associations. Because the analysis allowed for interstate variations (a plausible proxy for different market situations) differences in observed behavior seem to imply that stock associations were more aggressively seeking growth and income during the period studied, and were correspondingly less risk averse, than mutual associations.

A project being conducted jointly by Hester and Pierce is intended to provide a detailed dynamic structural description of financial intermediation by commercial banks and mutual savings banks. This project will ultimately draw together a number of related parts concerning a) rates of return earned by banks on different assets and liabilities, b) the relation between time paths of weekly deposit flows and portfolio composition, c) the predictability of deposits, d) bank profitability, e) simulation of individual bank behavior, and f) simulation of an aggregate monetary sector. To date,

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*A final report of this research is being completed for the Federal Home Loan Bank Board and will be publicly available in late 1968 as a contribution to a volume being edited by Professor Irwin Friend (University of Pennsylvania) for the Senate Banking and Currency Subcommittee.

encouraging results have been obtained* concerning the relation between portfolios and prior deposit flows, both for mutual savings banks and for commercial banks; concerning the predictability of demand deposits; and concerning the estimation of interest rates earned by banks.

One particularly interesting result is that it appears to take individual banks of both types about nine months to adjust fully to a random shock in deposits. Figure 4a illustrates the response pattern of a single commercial bank to a permanent $1 billion increase in demand deposits, as estimated from a large cross-section, time-series sample of New England commercial banks. (The curves reflect the pattern of interest rates and expectations existing at year-end 1963, and the initial portfolio of the particular bank used as a standard of reference. Apart from vertical displacements, however, the curves were assumed not to change over time.) The costs of portfolio adjustment partly explain the delay in acquiring less liquid assets. If the likelihood of imminent withdrawal of a deposit declines with its maturity, as found in another part of the study, uncertainty of deposit flows is also an explanation. Figure 4b shows the full adjustment of a hypothetical banking system, entirely consisting of such banks, in response to a $1 billion open market purchase of securities from the public by the Federal Reserve system. These results were generated by a simulation experiment in which all funds lent or invested by banks in the system in one month were assumed to be redeposited during the next month. About 90 per cent of the adjustment is completed in about 14 months, and it would take longer in a system with temporary leakages into institutions other than banks.

Another approach to bank portfolio analysis was used by Ritzmann in a study of Swiss banks, conducted while he was a visitor at the Cowles Foundation. Using cross-section data for 119 commercial banks at the end of 1959, he regressed banks' holdings of each asset on the structure of their liabilities; the hypothesis was that, given the structure of interest rates, the liquidity of a bank's asset portfolio would be related to the potential cash demands represented by its liabilities. The results of this phase of the study were corroborated in a canonical correlation analysis relating bank assets to bank liabilities. In this analysis, the coefficients of the variables, normalized to 1 for cash, represented a liquidity index for assets and liabilities. Thus, bills had a liquidity rating of .36 and mortgages of .13; demand deposits had a rating of .81, implying a self-imposed reserve requirement of about 20 per cent.

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*Preliminary results were reported by Hester at the 1965 Rome Econometric Society meeting (refer to [F] in the author's list) and by Pierce at the 1965 New York Econometric Society meeting.

FIGURE 4a   CHANGE IN A BANK'S ASSETS FOLLOWING
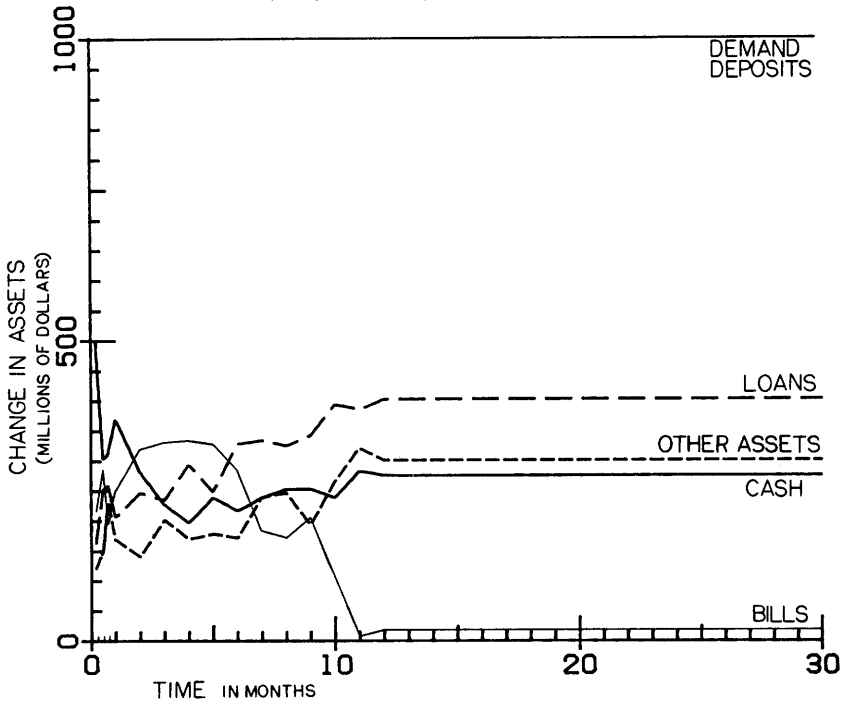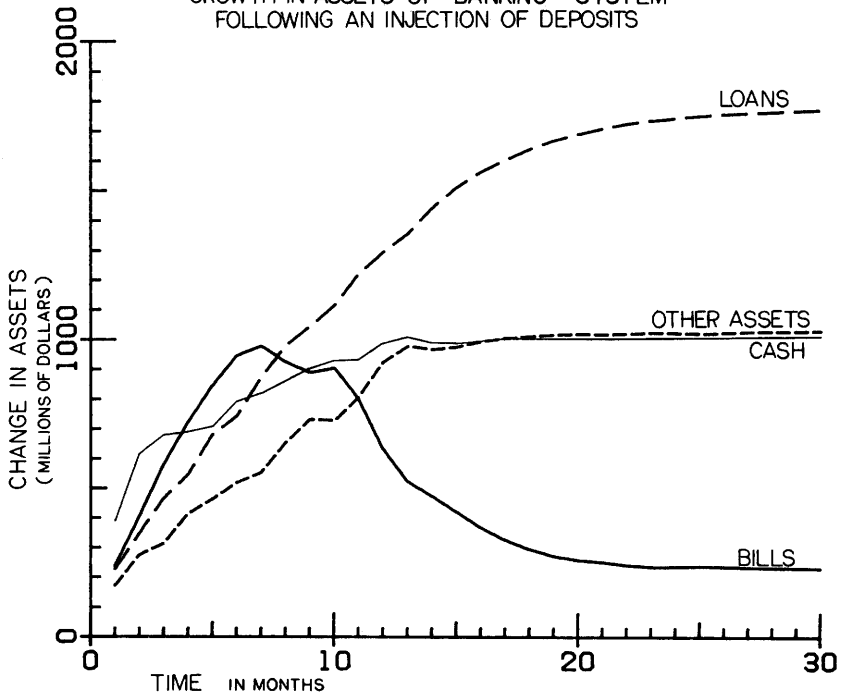AN INITIAL CHANGE IN DEPOSITS

FIGURE 4b   GROWTH IN ASSETS OF BANKING SYSTEM
FOLLOWING AN INJECTION OF DEPOSITS

AD. *General Equilibrium Involving Financial Markets.*

Analysis of the interaction of microeconomic portfolio decisions requires exploring the macroeconomics of general market equilibrium. As one step in this direction, Spencer (CFDP 223) developed an aggregate empirical econometric model of the U.S. savings and loan industry, that attempts to connect both markets in which saving and loan associations operate, the markets for share deposits and for mortgage loans.

Analysis of the interaction between monetary and real phenomena is potentially even more complex. One of the implications of the portfolio approach to monetary theory is to shift the emphasis away from those phenomena solely involving the quantity of currency and demand deposits and the operations of commercial banks. It is in the spirit of this approach that Tobin's review article (CFP 229) on the book by Friedman and Schwartz,* while recognizing the book's great achievement in historical scholarship, challenged the authors' high estimate of the causal importance of money alone in fluctuations of business activity.

An interesting question raised by the Friedman-Schwartz book and by other writings is the relevance of cyclical leads and lags in assessing causation. Does the fact that turning points in rates of change of the money stock precede those in money income or its rate of change lend credence to the claim that monetary changes are the causal force? Preliminary work by Tobin suggests that theoretical models where the money stock has no causal importance may generate observations with more conspicuous temporal precedence for monetary variables than some other models in which the money stock is all-important.

More complex models of the dynamic interaction of real and financial variables can best be explored and compared with the help of high-speed computers. Tobin and Brainard are engaged in simulation experiments to study the properties of various models of the financial system and of its linkages to the real economy. The simulation program, which they and a number of other investigators at Yale have been using, was written by Hester. Its key components are a simple algorithm for solving a system of non-linear equations and a matrix inversion procedure. This flexible Fortran IV program has also been used, in a number of graduate and undergraduate classes, to study simple static and dynamic models of income determination, to make forecasts on the basis of aggregative econometric models, and to study properties of one- and two-sector neoclassical growth models. In

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A Monetary History of the United States, 1867–1960, Princeton, Princeton University Press, 1963.

addition, the program is capable of generating Monte Carlo samples for studying various estimation problems and techniques.

BD. *Growth and Portfolio Balance.*

In a longer-run context, Tobin related portfolio theory and the theory of economic growth (CFP 234). In particular, he focused on the rate of growth of government monetary debt, the only financial asset available to the private sector that is not offset by an equal amount of private debt. He concluded that the equilibrium growth paths of the capital stock and of output depend on the rate at which this debt increases.

CD. *Rational Policy Under Uncertainty and the Theory of Teams.*

The analysis of rational decision making under uncertainty that has formed a cornerstone of portfolio theory is also applicable to the "theory of economic policy." A number of the familiar propositions of the theory of policy are based on the assumption that a policy maker knows with certainty the consequences of his own actions (or can find a "certainty equivalent"). For example, it has been argued that a policy maker needs only n instruments to attain n policy targets; if the number of instruments exceeds the number of targets, some may be left unused. In fact, however, one of the major problems confronting a policy maker is his difficulty in predicting the precise impact on target variables of the various instruments under his control.

Brainard analyzed (CFP 257) the implications of such uncertainty for the optimal conduct of policy. He found that, in a number of respects, the presence of such uncertainty qualitatively alters the conclusions derived in a world of certainty. For example, it is usually optimal to use all available instruments, even in trying to achieve only one target. Similarly, he found that, in general, it is not optimal to set policy so that the expected (i.e., mean) value of a target variable is equal to its target value.

One of the by-products of this analysis is a clarification of what is meant by the "effectiveness" of policy. Brainard has also been investigating the problems which uncertainty creates for the efficient decentralization of policy.

"Diversification" of industry has become a commonplace goal of economic policy in most less developed countries. One of the primary reasons for diversification, absent from the traditional model of international trade, is the uncertainty of returns associated with particular industries or products. In CFDP No. 197, Brainard and Richard Cooper (of Yale University)

show how the traditional model of international trade can be modified to take account of uncertainty in the price at which trade takes place. They show, as might be expected, that the presence of this type of uncertainty tends to reduce specialization. For a number of reasons, less developed countries may be concerned with the variance of their foreign exchange earnings. Brainard and Cooper discuss how the theory of portfolio choice can be used to explain differences in the variance of foreign exchange earnings across countries, and how this theory can help in the efficient selection of commodities to be imported or produced for export. They also discuss reasons for a divergence between private and social risk aversion, and various techniques which can be used to achieve socially optimal diversification.

A class of problems in decision making under uncertainty that are more of an operations research character are being studied by Kadane. These are problems of choosing a set or sequence of actions so as to accomplish an objective subject to some optimality criterion. One such problem (CFDP 220) has been formalized as choice of a strategy for searching for an object known to be present in one of n boxes with a given probability of success, and a given cost of search, for each box. The criterion for choosing searches turns out to be analogous to a benefit/cost criterion. The intent is to develop tools generally applicable to a broad and defined class of problems.

The theory of teams extends the analysis of decisions under uncertainty to the case of a team of decision makers acting in concert. Different members of the team may have access to different parts of the information pertinent to the outcome of action, whereas the transmission of information is costly. The theory of teams differs most from equilibrium theory, however, in that team theory assumes that all team members pursue exclusively a common goal. The theory that arises from these stipulations has been the subject of an investigation in depth by Jacob Marschak and Roy Radner, of the University of California at Los Angeles and at Berkeley, respectively, begun while both were associated with the Cowles Foundation. This work has now come to fruition in a forthcoming Cowles Foundation Monograph, *The Economic Theory of Teams*. The table of contents is reproduced on p. 49 below.

BCD. *Consumption Plans under Uncertainty.*

Risk affects consumers' optimal long-run saving plans as well as their portfolio choices. Two studies still in progress concern consumers' lifetime planning under conditions of uncertainty about future earnings. A study by Yaari is addressed to two questions: Is there a parameter of the distribu-

tion of future earnings, that might be called "expected permanent income" and could serve as a "certainty equivalent" for an uncertain income? Does the consumer's optimal policy converge as the life span considered becomes long? Preliminary results of this study were presented at a conference on the economics of uncertainty held at Harvard University in the summer of 1966.

In the other study, by Watkins, only two income levels are envisaged, corresponding to being employed and being unemployed. The individual describes his employment status by a Markov process. The particular question considered is the existence of a critical level of assets (a contingency fund) such that, if assets are below that level, the individual saves when he is employed.

ABD, ACD and ABCD. The "Empty" Cells.

Our Figure 1 is not intended as a catchall for all important categories of substantive economic research. It is blind to important lines of past, present and future research depending on distinctions other than the four, A, B, C, D, from which it is constructed. Even within the distinctions on which much of the Foundation's current empirical and theoretical research rests, however, there is scope for future syntheses that bring together all the elements now occurring separately or in smaller combinations. Indeed, it is tempting to speculate on the possible content of the as yet unfilled areas. ACD, policy oriented studies concerned with the interaction of many economic units making decisions under uncertainty, indicates a direction in which Tobin and Brainard are moving. As the interaction in financial markets becomes clearer, the efficiency of allocation resulting from these markets can be evaluated, and implications for financial regulations can be explored. Descriptive (ABD) and policy-oriented (ABCD) models of market equilibrium, in which economic agents make decisions under uncertainty of which the desired effects extend over time, would present an opportunity to bring together all the strands running through the studies described in this report.

E. Econometric Methods.

Research in statistical and econometric methods is reported under a separate heading even though, as Figure 1 suggests, these methods are pertinent to all the topics and categories already discussed. The reason for this separation is to maintain a sense of methodological unity in spite of diversity in areas of application.

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Another prefatory remark is that the work of Grether, Kadane, Nerlove and Wallis exhibits a natural continuity with the earlier work of the same staff members when connected with Stanford University, as faculty members and/or as students. For this reason, some references to that work, (as given in the lists of publications of each staff member, beginning on p. 56), will be included even though it was conducted at another institution.

1. *Time-Series Analysis in Economics*. Spectral techniques have an important use in the analysis of economic time series and, in particular, in the study of the problem of seasonality and of the effects of various methods of seasonal adjustment on the series to which they are applied. In the past, Nerlove, [B] and [C], devoted considerable effort to this field. Recently he has studied appropriate methods of estimating distributed lag relationships in the presence of serially correlated disturbances with relatively complex stochastic structure, such as that generated by the presence of seasonality. This work (CFDP 221) will appear in the forthcoming Fisher centennial volume. Important areas of application of such methods are in the study of sales-inventory relationships, and in that of short-run output-employment relationships. Work on the former topic has been carried out by Wallis (CFDP 209), and forms a portion of Grether's investigations described in more detail below. Work on the latter has been done by Hamermesh (CFDP 226) under Nerlove's supervision.

Problems of short-term forecasting are of considerable importance in the formulation of economic policy and in many areas of management science. One approach is to treat economic time-series as composed of several unobserved components with several different stochastic structures, and to use spectral methods to formulate appropriate models. An example appears in CFP 246 where Couts (Arthur D. Little, Inc.), Grether and Nerlove apply the approach to forecasting an employment series.

The notion that an economic time series may be divided into several unobserved components with different properties is an old one, going back at least to the work of Buys-Ballot in the first half of the nineteenth century. It continues to find application for instance, in the recent work on consumption of Milton Friedman of the University of Chicago. The early economic statisticians also practised the decomposition of economic time series into unobserved components; indeed the methods of seasonal adjustment developed by Persons, Frickey, Mitchell, and especially Kuznets, represent a crude form of this approach to the study of time series. The notion that components of different sorts in different series are related differently underlies most of the work of the National Bureau on business cycles and indicators. While simple and tentative, these ideas are of great importance

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in the formulation of economic policy and in business decisions, even though a majority of those working in econometric theory have neglected them. Grether is currently exploring the feasibility of treating the problem of decomposition of both single and multiple time series as one of signal extraction. A corpus of statistical theory, somewhat unfamiliar to economics, is available to be drawn upon for this purpose. In CFDP 202, Grether draws on some of the basic results of Peter Whittle of the University of Manchester to formulate the problem for a class of economically relevant models. The relation of this approach to the problem of distributed lags is explored in Nerlove's CFDP 221.

In addition to his work on inventories mentioned above, Wallis has also investigated various methods of estimating relationships containing lagged dependent variables, and having serially correlated disturbances.* In particular, Wallis compared the method of three-pass least squares suggested by Taylor and Wilson** with an alternative method which was computationally no more difficult. This method involved first estimating the serial properties of the disturbances by means of residuals calculated from the relationship estimated by the use of instrumental variables. Then these estimated serial properties were in turn used in a second round to improve the estimates of the parameters of the relationship in question. Wallis found that this method was superior to three-pass least squares, by a wide margin, in all the cases he considered.

Wallis also revised and modified substantially a computer program for spectral analysis of economic time series which had originally been written at Stanford University.***

2. *Treatment of Cross-Section Data and Cross-Sections Over Time.* In recent years there has been an accumulation of all sorts of cross-section data, much of which cover identifiable units over a period of time. Such data are potentially a much richer source of information than single cross-sections or aggregate time series. Present methods for the statistical analysis of cross-section, time-series data, particularly for the estimation of dynamic relationships, are in the beginning of their development, and the problems involved are as yet only partly understood.

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*"Lagged Dependent Variables and Serially Correlated Residuals: A Reappraisal of Three-Pass Least Squares," mimeo, July 27, 1966.

**Taylor, Lester D., and T. A. Wilson, "Three-pass Least Squares: A Method of Estimating Models with a Lagged Dependent Variable," *Review of Economics and Statistics*, November, 1964.

***"Description of a Computer Program for Spectral Analysis of Economic Time Series," mimeo., November 5, 1965.

For example, it has been found quite difficult to specify appropriate simultaneous-equations structures in cross sections, and to distinguish among variables which may be treated as predetermined and those which are jointly determined. The problem is less difficult, although still challenging, in dealing with production and cost functions for regulated industries such as electric power, natural gas, and local air service. Nerlove's work with Balestra on the demand for natural gas (CFP 255) suggested a number of alternative possible methods which might be used for estimation of dynamic behavior relations from a time series of cross sections. However, their results also posed a number of puzzling questions about the robustness of maximum-likelihood methods and the small-sample properties of several alternatives. Monte Carlo experiments that throw light on these questions will be reported by Nerlove in [K]. The methodology associated with time series of cross sections is also being tried out in the joint work of Hester and Nerlove on rates of return of common stocks and in the Hester-Pierce study, both described under D above.

Cross-section studies of production functions for unregulated industries, or of consumer behavior, will require a good deal of attention to the problem of simultaneous determination of the variables considered—in much the same way in which the problem of simultaneous relationships was examined in regard to time-series studies of supply and demand, or of consumption and investment, twenty to thirty years ago. In this vein, Nerlove has surveyed recent work on the identification and estimation of Cobb-Douglas production functions [J] and recent empirical studies of the constant-elasticity-of-substitution and related production functions [I].

3. *Formulation and Estimation of Multiple-Equation Econometric Models.* As the use of simultaneous-equations models, both sectoral and economy-wide, has become widespread in the past ten to fifteen years, the number of pertinent alternative estimation techniques has also grown. Comparison of these techniques has largely relied on their asymptotic properties as the sample size grows large. In his Ph.D. thesis at Stanford University,* Kadane developed a procedure for comparing estimators that applies to small samples as well. The procedure is based on Taylor expansions of the biases and the sampling variances and covariances of estimators with respect to a common numerical factor in the variances and covariances of all disturbances in the behavior equations. These expansions are useful for small values of that common factor, and are thus similar in spirit to the "theory of errors" of Gauss. Details have been worked out for all k-class estimators (which

*Presented at the annual meetings of the Econometric Society, December 1966; see [A] in author's list.

include ordinary least-squares, two-stage least squares, and limited-information maximum-likelihood estimators).

One interesting result from this study is that in equations in which the degree of over-identification is not larger than four, two-stage least-squares estimators have smaller variance than do limited-information maximum-likelihood estimators (for small values of the common factor). Since most Monte Carlo studies are conducted for small systems (e.g., CFP 226) and the results are applied to large systems, this result indicates that limited-information maximum-likelihood estimation could be wrongly discarded. Kadane's work is being extended to include the presence of lagged values of endogenous variables, to k-class estimators of the structure solved for reduced-form parameters, to the study of certain types of specification error, and to other estimators.

Hooper has approached the problem of simultaneous equations estimation from the standpoint of errors-in-variables models. His work shows that limited-information methods, in effect, determine the direction of minimization from the sample, whereas the two-stage least squares method fixes it *a priori.* Thus, while theoretically preferable, limited information may be less robust in the face of a specification error.

In estimating economic relationships, one usually obtains greater accuracy of estimation by incorporating information gained from previous samples or from economic theory. In fact, the very possibility of estimation often depends on the availability or assumption of such information. Over a number of years, including a period at the Cowles Foundation during the Spring of 1965, T. Rothenberg has examined the value of such *a priori* information in increasing the efficiency of parameter estimation. In addition to developing a general statistical framework for evaluating the efficiency gain (CFDP 205), he also applied the analysis to the simultaneous equations regression model. Explicit expressions were derived for the decrease in variance of reduced-form estimates which results from imposing overidentifying restrictions on the structural relationships (CFDP 213). This research, which initially was the basis of an MIT doctoral dissertation, has since been extended, and is slated to appear as a Cowles Foundation Monograph.

GUESTS

The Cowles Foundation is pleased to have as guests scholars and advanced students from other research centers in this country and abroad. Their presence contributes stimulation and criticism to the work of the staff and aids in spreading the results of its research. To the extent that its resources permit, the Foundation has accorded office, library, and other research facilities to guests who are in residence for an extended period. The following visited or were associated with the organization in this manner during the past three years.

SYDNEY S. AFRIAT (Rice University), July 1964–January 1965. Sponsored by the National Science Foundation.

DAVID I. FAND (State University of New York at Buffalo), August 1964–August 1965. Sponsored by the National Science Foundation and the State University of New York at Buffalo.

GERALD H. KRAMER (University of Rochester), July 1966–July 1967. Sponsored by the National Science Foundation and the University of Rochester.

FRANZ RITZMANN (Institute for Economic Research [Wirtschaftswissenschaftliches Institut], University of Zurich, Zurich, Switzerland), September 1965–June 1966. Sponsored by the University of Zurich and Yale University.

AGNAR SANDMO (Norwegian School of Economics and Business Administration, [Norges Handelshöyskole], Bergen, Norway), September 1964–June 1965.

JAN SERCK-HANSSEN (Institute of Economics, University of Oslo, Oslo, Norway), September 1964–July 1965. Sponsored by the Rockefeller Foundation.

JOHN E. SPENCER (The Queen's University of Belfast, Belfast, N. Ireland), September 1965–June 1966. Sponsored by Henry Fellowship.

LESTER G. TELSER (Graduate School of Business, University of Chicago), September 1964–June 1965. Sponsored by the National Science Foundation.

EDWARD ZABEL (University of Rochester), September 1964–July 1965. Sponsored by the Ford Foundation.

KARL H. BORCH (Norwegian School of Economics, Bergen, Norway, and Nuffield College, Oxford). Visited in April 1967.

COWLES FOUNDATION SEMINARS

In addition to periodic Cowles Foundation staff meetings, at which members of the staff discuss research in progress or nearing completion, the Foundation also sponsors a series of Cowles Foundation Seminars conducted by colleagues from other Universities or elsewhere in Yale. These speakers usually discuss recent results of their research on quantitative subjects and methods. All interested members of the Yale community are invited to these Cowles Foundation Seminars, which are frequently addressed to the general economist including interested graduate students. The following seminars occurred during the past three years.

July 1, 1964–June 30, 1967

1964

October 30. PIETER DE WOLFF, Central Planning Bureau, The Hague, and Harvard University, "An Optimal Price Policy for Natural Gas Deposits."

November 30. J. R. N. STONE, Cambridge, England, "A Model of the UK Educational System and its Contribution to Economic Growth."

December 11. DAVID GALE, Brown University, "Optimal Operation of a Multi-sector Economy."

December 17. PETER A. DIAMOND, University of California, Berkeley, "National Debt in a Neoclassical Growth Model."

1965

January 5. MARC NERLOVE, Stanford University, "Applications of Spectral Techniques in Economics."

January 22. ALLAN H. MELTZER, University of Chicago and Carnegie Institute of Technology, "The Interaction of Money, Credit and Interest Rates on the Bank-Oriented Credit Market."

February 12. IRVING LAVALLE, Harvard Business School, "A Bayesian Approach to Game Problems."

April 13. LAWRENCE R. KLEIN, University of Pennsylvania, "Non-Linear Stochastic Models."

May 7. HIROFUMI UZAWA, University of Chicago, "A Monetary Model of Economic Growth."

June 3. LAWRENCE FISHER, University of Chicago, "Two New Sets of Common Stock Indexes."

November 19. HENDRIK S. HOUTHAKKER, Harvard University, "The Dynamics of Consumption and Savings."

1966

January 7. JAMES DURBIN, Johns Hopkins University and London School of Economics, "A New Look at the Problem of Testing for Serial Correlation in Least-Squares Regression."

January 28. FRANK BRECHLING, Massachusetts Institute of Technology, "Short-Run Production Functions: An International Comparison."

February 11. RALPH GOMORY, IBM Watson Research Center, "On the Relation between Integer and Non-Integer Solutions to Linear Programs."

April 13. GEORGE DANTZIG, University of California at Berkeley, "Complementary Theory in Mathematical Programming."

April 29. HAROLD W. WATTS, Office of Economic Opportunity and University of Wisconsin, "The Iso-Prop Index: An Approach to the Determination of Poverty Income Differentials."

May 10. SHINICHI ICHIMURA, University of California at Berkeley and Osaka University, "An Econometric Model of the Monetary Sector of Postwar Japan."

May 27. GUY ORCUTT, Harvard University and University of Wisconsin, "The Effects of Aggregation on Estimation."

December 9. LEONID HURWICZ, University of Minnesota, "On Problems of Economic Organization."

1967

February 17. ALAN A. WALTERS, University of Birmingham and Massachusetts Institute of Technology, "A Simple Model of an Uncongested Road."

March 3. ZVI GRILICHES, University of Chicago, "Sources of Measured Productivity Growth—The Current State of Research on the Residual."

March 17. CARLTON E. LEMKE, Rensselaer Polytechnic Institute, "Mathematical Programming and Bi-Matrix Games."

April 7. RICHARD N. ROSETT, University of Rochester, "The Experimental Measurement of Subjective Probability and its Relation to Relative Frequency."

April 14. KARL BORCH, Nuffield College, Oxford, and The Norwegian School of Economics and Business Administration, Bergen, "The Theory of Risk."

May 5. MICHAEL C. LOVELL, Carnegie Institute of Technology, "Firm Sales, Anticipations, Planned Inventory Behavior, and the Production Decision."

May 8. EDMOND C. MALINVAUD, University of California, Berkeley, and Ecole Nationale de la Statistique et de l'Administration Economique, Paris, "The Theory of Non-Linear Regression."

May 26. W. M. GORMAN, Stanford University and Oxford, "On the Structure of Separable Functions."

FINANCING AND OPERATION
OF THE COWLES FOUNDATION

The Cowles Foundation relies largely on gifts, grants and contracts to finance its research activities. Yale University contributes to the Cowles Foundation the use of a building at 30 Hillhouse Avenue which provides office space, two seminar rooms, and other facilities. The University also supports the Foundation's research and administration through paying or guaranteeing part or all of the non-teaching fractions of the salaries of three permanent staff members.

The gifts of the Cowles family are the cornerstone of Cowles Foundation financial support. These gifts provide a permanent source of untied funds that assure the permanent staff continuing research support, that permit the staff freedom to shift the balance of their time among various subjects of research, and that provide for general operating expenses not appropriately chargeable to grants and contracts for work on specific topics. In addition, a growing amount of financial support has come from grants and contracts (primarily short-term) from the National Science Foundation, the Office of Naval Research, and private foundations.

The major part of Cowles Foundation expenditures is accounted for by research salaries (and associated fringe benefits). The rest of the budget consists of office and library salaries, overhead expenses such as the costs of preparing and distributing manuscripts, and some of the costs of computing services (the Yale Computer Center currently makes available the services of a direct coupled IBM 7040-7094 system, an IBM 1401, and necessary auxiliary equipment).

The pattern of Cowles Foundation income and expenditures in recent years is outlined in the table below.

ANNUAL INCOME AND EXPENDITURES
OF THE COWLES FOUNDATION

Average for		INCOME				EXPENDITURES		
	Total	------ Permanent ------ Cowles Family Gifts	Yale	Total	Temporary— mostly project support	Total	Research Salaries	Other
1961–64								
$(000)	179	41	12	53	126	180	112	68
%	100	22.9	6.7	29.6	70.4	100	62.2	37.8
1964–67								
$(000)	250	44	14	58	192	244	148	96
%	100	17.6	5.6	23.2	76.8	100	60.7	39.3

During the period of this report, the size of the research staff at the Cowles Foundation fluctuated, as it had in earlier periods. The size of the

staff has depended largely on the opportunities that have developed for bringing to the Foundation—usually on a joint appointment with the Department of Economics or another department—colleagues interested in the development and application of mathematical and quantitative methods in economics (or other social sciences), or inviting a visitor or a colleague already on the Yale faculty to share in research activities in the Cowles domain. The research staff in the past three years has included six permanent members, consisting of five professors in the Departments of Economics, and one in that of Administrative Sciences, and from 9 to 13 faculty members on term appointments, who are members of those departments or of the Department of Statistics. On average, both the permanent and the younger members of the staff devote about half of their professional effort in the academic year, and up to two full months in the summer, to their research and to interaction with their colleagues.

These activities are supported by the services of five secretaries and manuscript typists who, under the direction of Miss Althea Strauss, prepare and circulate Cowles Foundation Papers and Discussion Papers. A varying number of student research assistants and a part-time computer programmer, Mrs. Elizabeth Boeckelman, assist directly in the research studies.

A small library, under the supervision of Mr. Vaughn Simon, is maintained in the building of the Cowles Foundation. It makes research materials readily available to the staff and supplements the technical economics and statistics collections of other libraries on the Yale campus. The collection includes a permanent collection of some 4,700 books and 170 journals primarily in the fields of general economics, mathematical economics, econometric studies and methods, statistical methods and data; numerous pamphlets from Government sources and international organizations; series of reprints from 22 research organizations at other Universities in the United States and abroad; and a rotating collection of recent unpublished material. About 650 of the books in the collection were acquired in the past 3 years, almost all of which had been published just shortly before acquisition. During the same period, the library was able to dispose of about 400 books (either to other libraries at Yale or by sale to dealers) that had become less pertinent to current research activities of the Cowles Foundation staff. Although the library is oriented primarily to the needs of the staff, it is also used by other members of the Yale faculty and by students of the University. About 250 books which are in demand for graduate courses are kept on reserve and circulate overnight and weekends. All other books circulate for periods up to a month to anyone affiliated with the University. Journals may be borrowed by the staff, and may be used by anyone else in the library.

MONOGRAPHS
1934–1967*

The monographs of the Cowles Commission (Nos. 1–15) and Cowles Foundation (Nos. 16–21) are listed below:

No. 1. *Dynamic Economics*, by CHARLES F. ROOS. 1934. Evanston, Ill.: Principia Press. 275 pages. (Out of print.)

No. 2. *NRA Economic Planning*, by CHARLES F. ROOS. 1937. Evanston, Ill.: Principia Press. 596 pages. (Out of print.)

No. 3. *Common-Stock Indexes*, by ALFRED COWLES and ASSOCIATES. Second Edition, 1939. Evanston, Ill.: Principia Press. 499 pages. Price $8.15. Monthly indexes of stock prices, stock prices adjusted for reinvestment of cash dividends, and yield expectations; and annual indexes of yields, divided payments, earnings-price ratios, and earnings for 69 industry groups, 1871–1938.

No. 4. *Silver Money*, by DICKSON H. LEAVENS. 1939. Evanston, Ill.: Principia Press. 439 pages. A sketch of the history of the monetary use of silver, followed by more detailed consideration of recent developments. (Out of print.)

No. 5. *The Variate Difference Method*, by GERHARD TINTNER. 1940. Evanston, Ill.: Principia Press. 175 pages. The history and use of this method for the analysis of time series, with new devices of treatment and extensive tables to aid calculations. (Out of print.)

No. 6. *The Analysis of Economic Time Series*, by HAROLD T. DAVIS. 1941. Evanston, Ill.: Principia Press. 620 pages. The historical development of the subject is reviewed, methods are described, and applications made to economic phenomena. (Out of print.)

No. 7. *General-Equilibrium Theory in International Trade*, by JACOB L. MOSAK. 1944. Evanston, Ill.: Principia Press. 187 pages. The modern theory of economic equilibrium (as stated by J. R. Hicks and others) applied to an important field. (Out of print.)

No. 8. *Price Flexibility and Employment*, by OSCAR LANGE. 1944. Evanston, Ill.: Principia Press. 114 pages. Price $4.05. A clarification of important concepts that have had much currency in the practical discussion of depression and of economic stabilization generally but had yet to be formalized in a complete theoretical structure.

No. 9. *Price Control and Business*, by GEORGE KATONA. 1945. Evanston, Ill.: Principia Press. 246 pages. A study of the working of price control based on field studies among producers and distributors of consumers' goods in the Chicago area, 1942–1944. Price $4.40.

No. 10. *Statistical Inference in Dynamic Economic Models*, edited by TJALLING C. KOOPMANS, with Introduction by JACOB MARSCHAK. 1950. New York: John Wiley and Sons. 438 pages. Prices $7.50. Original contributions from many authors concerning statistical problems encountered in economic model construction.

No. 11. *Economic Fluctuations in the United States, 1921–1941*, by LAWRENCE R. KLEIN. 1950. New York: John Wiley and Sons. 174 pages. Price $4.50. The methodology of econometric model construction is applied to business cycle analysis with possible implications for prediction and policy making.

*Orders for Monographs 3, 4, 8, and 9 should be sent to Trinity University Press, 715 Stadium Drive, San Antonio, Texas. Orders for subsequent monographs should be sent to John Wiley and Sons, 605 Third Avenue, New York City. Prices are subject to change.

No. 12. *Social Choice and Individual Values*, Second Edition, by KENNETH J. ARROW. 1963. New York: John Wiley and Sons. 124 pages. Price $4.50. Presents the original text on the theory of social choice with an appended commentary containing a series of reflections on the text and on some of the more recent literature.

No. 13. *Activity Analysis of Production and Allocation*, edited by TJALLING C. KOOPMANS. 1951. New York: John Wiley and Sons. 404 pages. Price $5.50. Contributions from economists and mathematicians on the theory and techniques of efficient allocation of resources and programming of activities.

No. 14. *Studies in Econometric Method*, by COWLES COMMISSION RESEARCH STAFF, edited by WILLIAM C. HOOD and T. C. KOOPMANS. 1953. New York: John Wiley and Sons. 324 pages. Price $7.95. Presents and extends methods developed in Monograph 10 in an expository style addressed primarily to the user of methodology.

No. 15. *A Statistical Study of Livestock Production and Marketing*, by CLIFFORD HILDRETH and F. G. JARRETT. 1955. New York: John Wiley and Sons. 156 pages. Price $6.50. Economic relations underlying the operation of livestock markets in the United States are estimated and tested by several alternative procedures.

No. 16. *Portfolio Selection, Efficient Diversification of Investments*, by HARRY M. MARKOWITZ. 1959. New York: John Wiley and Sons. 344 pages. Price $7.50. Presents methods for translating anticipations about future yields of securities, and about their interrelations, into investment decisions that give minimum expected risk for given expected return.

No. 17. *Theory of Value, An Axiomatic Analysis of Economic Equilibrium*, by GERARD DEBREU. 1959. New York: John Wiley and Sons. 114 pages. Price $5.50. A rigorous presentation of the theories of producers' behavior, consumers' behavior, Walrasian equilibrium, Paretian optimum, and of their extensions to uncertainty.

No. 18. *Studies in Process Analysis: Economy-Wide Production Capabilities*, edited by ALAN S. MANNE and HARRY M. MARKOWITZ. 1963. New York: John Wiley and Sons. 427 pages. Price $14.00. Methods for estimating the production capabilities of an economy or an industry, using engineering information and mathematical programming techniques. Applications to petroleum, chemical, metal and metalworking industries, to agriculture, and to economic development. Includes spatial aspects of production.

No. 19. *Risk Aversion and Portfolio Choice*, edited by DONALD D. HESTER and JAMES TOBIN. 1967. New York: John Wiley and Sons. 180 pages. $8.95.

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No. 20. *Studies of Portfolio Behavior*, edited by DONALD D. HESTER and JAMES
TOBIN. 1967. New York: John Wiley and Sons. 258 pages. $8.95.

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No. 21. *Financial Markets and Economic Activity*, edited by DONALD D. HESTER
and JAMES TOBIN. 1967. New York: John Wiley and Sons. 256 pages. $8.95.

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SPECIAL PUBLICATIONS

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