REPORT *of* RESEARCH ACTIVITIES

July 1, 1958–June 30, 1961

COWLES FOUNDATION
FOR RESEARCH IN ECONOMICS
AT YALE UNIVERSITY

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PURPOSE AND ORIGIN

THE COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY, *established as an activity of the Department of Economics in 1955, has as its purpose the conduct and encouragement of research in economics, finance, commerce, industry, and technology, including problems of the organization of these activities. The Cowles Foundation seeks to foster the development of logical, mathematical, and statistical methods of analysis for application in economics and related social sciences. The professional research staff are, as a rule, faculty members with appointments and teaching responsibilities in the Department of Economics and other departments.*

The Cowles Foundation continues the work of the Cowles Commission for Research in Economics, founded in 1932 by Alfred Cowles at Colorado Springs, Colorado. The Commission moved to Chicago in 1939 and was affiliated with the University of Chicago until 1955. In 1955 the professional research staff of the Commission accepted appointments at Yale and, along with other members of the Yale Department of Economics, formed the research staff of the newly established Cowles Foundation.

The Econometric Society, an international society for the advancement of economic theory in its relation to statistics and mathematics, is an independent organization which has been closely associated with the Cowles Commission since its inception. The headquarters of the Society were moved from Chicago to Yale in 1955.

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS AT YALE UNIVERSITY

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NOTE ON REFERENCES TO PUBLICATIONS

The following abbreviations are used throughout this report in referring to publications or working papers of the Cowles Foundation and Cowles Commission:

CCNS: Cowles Commission New Series Papers (see p. 53)

CFP: Cowles Foundation Papers (see p. 50)

CFDP: Cowles Foundation Discussion Papers (see p. 56)

Monographs (see p. 47) are referred to by number, and *Special Publications* (see p. 49) by title.

Other publications of each staff member are designated by letter in the list on pp. 59-61 and are referred to by author and letter in the text.

RESEARCH ACTIVITIES

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INTRODUCTION

TAKING the long view, perhaps the principal element of continuity in the research of the Cowles Foundation (and of its predecessor the Cowles Commission) is its identification with econometrics. What is econometrics? It consists of the design and use of rigorous formal methods of logic, mathematics, and statistical inference for application to economic theory, to the testing of economic hypotheses, and to the estimation of economic relationships. The common commitment to such methods is what gives unity and identity to the work of the Cowles Foundation.

A focus of this kind allows great diversity in the substantive topics of individual pieces of research. Over the years, along with increasing availability and acceptance of a variety of econometric methods and a richer flow of data, the substantive diversity of econometric research has increased, in the profession at large as well as in the activities of the Cowles Foundation. Yet it occasionally happens that, without specific planning to that effect, a broader common substantive interest becomes apparent in several independently conceived studies. In our research of the last few years, one particular aspect of economic choice has turned out to be a common thread of a number of studies, both theoretical and empirical.

The individual studies in question were addressed to such widely different problem areas as timing preference, economic growth, capital formation, saving and consumption, intergeneration transfers, choice of assets, and monetary theory and policy. The common thread that is found to run through all these pieces of research is a concern with the bridging of time in economic choice. How does the individual make his choices between present and future rewards for his efforts, or between present and future returns on his assets? Where such a choice is made by or for an entire society, how can it be made consistently in all the detailed decisions bearing on that choice?

Studies in which the bridging of time is an important element will first be reported on together, proceeding from the abstract and theoretical to the empirical and concrete. Thereafter, the report proceeds to a variety of other studies in which the element of intertemporal choice is less central or conspicuous, though rarely absent: studies of economic equilibrium in

competition between many or few, of production capability, of inventory fluctuations and economic forecasting, of bank lending, of international trade, of labor mobility, of managerial economics, of organization theory, and of decision-making under uncertainty. The report concludes with a description of research concerned with the development of statistical and mathematical tools and methods of econometrics and of economic theory.

For support of the research reported below, the Cowles Foundation is indebted to a variety of donors. The firm financial basis for the Cowles Foundation's continuing activity is provided by Alfred Cowles and other members of the Cowles Family, and by the University. Additional general support for the research program is being given through a five-year grant by the Rockefeller Foundation. The work in organization theory, in decision-making, in managerial economics, on economic equilibrium, and on the choice of productive techniques in economic development, is supported by the Office of Naval Research. Specific grants from the National Science Foundation have supported a Monte Carlo study by Summers of certain methods of estimating economic behavior constants, work by Koopmans on stationary utility and on development paths arising from its maximization, and the initial phase of the work by Srinivasan on choice of production techniques in economic development.

Timing Preference

The idea that most people, against a background of an assured even flow of consumption over time, would prefer a given additional benefit to come in the near future rather than in a more distant future, has a long history in economic thought. It was first put forward by von Böhm Bawerk, and developed further by Irving Fisher. The same idea was examined from a more abstract point of view in a study by Koopmans (CFP 151), which was continued, first with the assistance of Diamond, and thereafter during Koopmans' year of leave at Harvard University in cooperation with Richard Williamson, mathematician at Dartmouth College. In order to avoid imposing an arbitrary termination date on what actually is an indefinite, open-ended, future, this study considers preference—to be interpreted as social rather than individual preference—among consumption programs for an *infinite* future. Each such program is itself an infinite sequence of consumption bundles imagined available with certainty in successive future periods. The two key postulates of the study are (a) that this preference is expressible by a utility function with certain continuity properties, and (b) that while preference may depend

in some way on the lengths of time to elapse between the time of choice and the times of availability of the consumption bundles in prospect, preference is not otherwise dependent on the time of choice itself. The term *stationary utility* was used to express these properties. With the help of three further postulates that seem acceptable as a basis of inquiry it was found that preference for early timing of future benefits, or *impatience* as Fisher has called it, is a logical consequence of the postulates. In the context of this study, therefore, impatience is no longer an observed psychological trait of most people. Neither, for that matter, is it an ethical principle in balancing the opposed interests of present and future generations—nor a violation of such a principle. It is simply an unexpected but inescapable implication of reasonable postulates which when considered individually appear to say nothing about timing preference. It is the open-endedness of time that introduces the "paradox," if paradox it is.

Choice of Development Paths

To simplify matters further, imagine now that only a single good enters into consumption in all future periods. Imagine further an extremely simple technology in which, starting with a given initial stock of that good, that part of the stock not consumed in the first period grows at a constant "technological" growth rate to form the initial stock of the second period, and so on. In an economy that simple, consumption and investment are one single decision. What will be the outcome if a program is to be chosen under these circumstances in such a way that its utility is maximized?

Preliminary results of Koopmans' work on this problem indicate that the answer depends critically on the "subjective" interest rate implied in the utility function being maximized. To define this interest rate compare a program with a constant consumption flow for all future periods with two slightly better programs, one with only the first-period consumption somewhat increased, the other with only the second-period consumption increased. If these increments are small and so chosen that the two better programs are equally desirable, then the percentage excess of the increment in consumption specified for the second period over that specified for the first period is the subjective interest rate in question. It is a numerical measure for the degree of impatience.

Now if there exists a constant consumption flow such that for that flow the subjective interest rate equals the given technological growth rate, *then* an initial stock that just makes that constant consumption flow

possible will actually cause that flow to be chosen through utility maximization. A larger initial stock leads to a larger consumption flow in each future period, but not necessarily larger by the same amount in each period. As time goes on, the increment in consumption due to the increased initial stock will (a) tend to zero or (b) become larger and larger, depending on whether, for a slightly larger *constant* consumption flow, the subjective rate of interest is (a) larger or (b) smaller than for the original constant flow.

It is hoped that the study of highly simplified and artificial models of this kind will throw some light on the much more complicated realities of economic development, and perhaps also suggest concepts and tools with which to approach these realities. Such is also the motivation of Srinivasan's doctoral dissertation (Yale 1961), a study of criteria for choice from a constant collection of techniques of production in economic development. While total saving and total investment still remains in some sense a single decision, this study concentrates on the choice of the particular investment good or goods to be produced at each time. In Srinivasan's model a single consumer good can be produced by each one of an infinite sequence of techniques, which are labeled in order of increasing capital requirements, and simultaneously in order of decreasing labor requirements. Capital, on the other hand, is produced by a single method using capital and labor, and is subject to depreciation at a constant rate regardless of use. The labor force is assumed to grow at a constant proportionate rate. It is shown that the economy considered is capable of generating a maximum sustainable rate of consumption per worker. This rate is attained at each point of time along a "terminal path" in which only one most productive technique of producing the consumer good is used. This path is reached at such time at which every worker in the consumption goods industry is supplied with the full amount of capital required by that technique. Since that amount is not initially available in the actual circumstances of developing countries, approaches to the "terminal path" from an initial position not on it are also considered. In particular, the path which minimizes the time needed to reach the "terminal path" from below is derived. It is shown further that the growth paths that result from applying some of the investment criteria proposed in the literature are likewise of an extreme character. It is suggested by this analysis that the problem of formulating simple operational criteria for the choice of technique in economic development is still unsolved. A somewhat more general problem of characterizing the class of all efficient growth paths is also discussed by Srinivasan (CFDP 117).

Chenery made a critical survey (CFP 161) of the literature on development planning for a underdeveloped countries, in order to evaluate the various criteria for allocating investment funds and other scarce resources. In particular, he contrasted and compared the criteria suggested by the traditional doctrine of comparative advantage in the international division of labor, and by more recent theories of growth, especially balanced growth. He observed that the comparative advantage point-of-view has not recognized differences between prices of the factors of production and the real opportunity costs of their use, changes over time in quantity and quality of factors, increases in productivity obtainable by drastic increases in the scale of production, and interdependence of production processes. On the other hand, modern growth theories have tended to overlook the very real advantages from international trade stressed by classical authors.

Chenery suggests "activity analysis" (or "linear programming") models as a suitable framework through which to meet and overcome most of the objections to both approaches and to combine their points of view. However, the advantages of large-scale production are not expressible in such models without substantial modification.

Finally, his study discusses planning procedures in use in a number of countries in the light of this analysis.

The effect of scale of production on productivity, emphasized by Chenery, is central to a study by Manne addressed primarily to the development planning of the firm. This study (CFDP 54, revised) concerns the optimal degree of excess capacity to be built into a new facility such as a pipeline, a steel plant, or a super highway, if the facility will be long-lived while demand is expected to grow strongly over time. This study considers both the case where the growth of demand is foreseen with certainty, and the case where a certain randomness in the growth of demand is assumed. In the latter case the question is examined in which way the degree of uncertainty concerning future demand affects the optimal capital expansion policy, and the expected cost of production incurred under that policy. It is found, as one might anticipate, that greater uncertainty raises cost, but also, less readily foreseen, that greater uncertainty calls for larger facilities to be added when expansion is due, and hence for a higher average over time of excess capacity. These findings apply to the case where unsatisfied demand is lost forever. Modifications are indicated for the case in which a backlog of unsatisfied demand can be carried over into subsequent periods.

Beckmann studied (CFDP 120) the growth path resulting from a

monetary policy that holds the interest rate fixed. Other assumptions specify a production function with no effects of scale on productivity, and a constant rate of population growth. These assumptions are embedded in a "static" Keynesian model made "dynamic" by including the percentage rate of change in the price level among the determinants of investment, consumption, money supply, and real money demand functions. It is found that initial unemployment leads to an unstable process of balanced growth. From initial full employment a Wicksellian cumulative process develops: There exists a natural rate of interest such that at interest rates below the natural rate, balanced growth is attained and the price level rises; at interest rates above the natural rate both employment and the price level are falling. The "optimal" interest rate which maximizes per capita consumption at any time during balanced growth is shown to equal the rate of growth of population.

The Impact of Technical Progress on Growth

Most of the foregoing studies presuppose a given technology. New dimensions of economic growth are opened up if one recognizes technological progress. In fact, recent quantitative studies of growth in the United States have led economists to downgrade the effectiveness of investment in raising output per worker, and accordingly to revise upward the effect of technological change. Still, if there were no fresh investments to embody technological advances, productivity would grow hardly at all. The modernizing effect of investment was formalized by Robert Solow of the Massachusetts Institute of Technology in an aggregative model of production. In this model the substitutability of capital (of any given vintage) for labor and conversely is described by a so-called Cobb-Douglas production function.

In order to trace in greater detail the implications of this "new" view of investment, Phelps (in CFDP 110) compared two models of growth, having the same fraction of income invested, the same rate of depreciation on capital, and the same Cobb-Douglas model of substitution between labor and capital. The two models also have the same annual percentage rate of increase in output, from given capital and labor inputs, resulting from technical progress. However, in one model, representing the "old" view, output is determined on the basis of technical progress up to the time of the production in question, whereas in the other model, reflecting the "new" view, only technical progress up to the time of construction of the capital used in production is taken into account.

It was found that in the long run output from a given labor input is no more sensitive to investment in the "new" model than in the "old" model. This is because the long-run average age of the capital stock depends only upon the average growth rate and the depreciation rate and neither, in these Cobb-Douglas models, depend upon investment policy in the long run. However, the new model grants a greater role to investment in the short run. Given the fraction of output to be saved, output is quicker to approach its long-run equilibrium growth path in the new model than in the old model. In that sense the new model offers a higher return to increased thrift. Of course, the two models will not interpret present conditions in the same way, hence need not forecast the same output path corresponding to a given investment policy.

In work now in progress, Phelps is attempting to measure the importance of some of the factors in the growth of selected economies in Europe and North America since the war. This work involves aggregate and sector econometric models in which investment serves both to deepen and modernize the capital stock.

Saving and Consumption

We return to the total savings decision, and look on it now as a resultant of the savings decisions of individual households and other decision units. Phelps has considered what is the effect, on the household's desire to save, of uncertainty as to the future market value of the accumulated assets. One model developed to study this problem (CFDP 101) resembles Ramsey's in "A Mathematical Theory of Saving": consumption is continuous through time. But in the present paper, unlike Ramsey's, capital is subjected to randomly timed gains and losses. Some of the consequences of this capital risk are the following: The optimum consumption rate does not depend simply upon the expected income flow. To the consumer with risk aversion, the riskiness of income (capital) makes a difference. In the untruncated (horizonless) and undiscounted case (no pure time preference) the presence of capital risk produces a smaller consumption rate than is found in the comparable limiting "riskless" case. Ramsey's conclusion that the optimum saving rate is independent of the interest rate when future utility is not discounted fails to carry over to the present case.

The same problem is attacked in another model (CFDP 109) in which consumption decisions and capital growth (or loss) occur at periodic intervals rather than continuously. The rate of growth of unconsumed

capital from period to period is a random variable. Each period the household consumes a portion of its capital (including its wages) and the capital remaining is left to grow or, worse luck, decay. Some of the results of this analysis are: Consumption is an increasing function of the household's capital and the household's age. The "hump saving" phenomenon, where the individual's net worth reaches a maximum at some point in middle age if everything goes according to plan, may not arise if capital is rather risky, despite its net expected productivity. The proposition that risky income has a smaller impact upon consumption than certain income is supported by certain examples. Just as the effect of variations in the rate of return on capital upon the propensity to consume is indeterminate without some restrictive assumptions on the utility function, so too the effect of variations in the degree of risk depends upon the shape of the utility function.

A similar study by Beckmann (CFDP 68) deals with the opposite case, where the individual expects to carry his assets into the future without risk, but where the income from his labor is subject to uncertainty. Let income be a random variable independently and identically distributed in all periods of an indefinitely long future, and let the utility function for current consumption be unchanging through time, whereas future utilities are discounted at constant compound interest rates. If wealth, including accumulated savings, yields a riskless return at a constant market rate of interest, the optimal allocation of the consumer's income to saving and consumption is a function of wealth only. To determine it, one expresses the utility of wealth as the maximum sum of the utility of consumption in the current period and of the discounted expected value of the utility of wealth at the end of the current period. The shape of the utility-of-wealth function thus defined is concave, whenever the utility-of-consumption function is concave. It follows that consumption is a non-decreasing function of wealth. The consumption level changes with shifts in the expected value of income and also responds to temporary increments (windfalls), but the former effect is substantially larger.

A considerable amount of empirical work bearing on the same group of questions was done during the period of this report. The research on the structure and dynamics of household balance sheets which was described in the 1956-58 report was completed in early 1959 and resulted in a joint paper by Watts and Tobin. This paper was presented at The Conference on Consumption and Saving held at The Wharton School on March 30-31, 1959, and was subsequently published (CFP 165) in Vol.

supports the notion that there is an "average" or "preferred" portfolio papers presented at the Conference. The evidence provided by this study II of *Consumption and Saving,* edited by Irwin Friend, along with other of assets and debts for a household which varies among households according to their economic and social circumstances. An analysis of the financial flows of households, such as durable goods purchases, debt repayment or acquisition, saving in various forms, was also made. This analysis disclosed that the flows in each period tend to eliminate discrepancies between the "average" or "preferred" portfolios and the household's actual portfolio at the beginning of the period.

The 1950 Survey of Consumer Expenditures made by the Bureau of Labor Statistics, which provided the data for the Watts-Tobin paper, has also provided data for three studies by Watts to test hypotheses put forward by Milton Friedman of the University of Chicago. In the first of these studies an attempt was made, by using the method of instrumental variables, to identify and obtain estimates of the elasticity of consumption with respect to transitory or temporary income. One of Friedman's hypotheses asserts that this elasticity is equal to zero. Unfortunately the assumptions required to test this hypothesis are quite stringent, and are apparently not met by variables employed in the analysis. To the extent that any conclusion is warranted by the evidence produced, it is that the assumption of zero elasticity is extreme. On the other hand, it is clear that consumption is less sensitive to "transitory" income changes than to "permanent" income changes. Some of the other parameters of Friedman's model were estimated also, and, if internal consistency and reasonableness can be used as criteria, these estimates are less sensitive to departure from the assumptions made in choosing the estimating procedure.

The second of these studies culminated in a paper by Watts (CFDP 99) in which an operational permanent income variable is defined and given a tentative trial to evaluate its explanatory ability. This variable makes a household's permanent income a function of the average income-age relation (income profile) of "similar" households together with the deviation of the household's current or recent income from that profile. The definition of this variable depends on two parameters. One is a discount factor which specifies the declining schedule of weights applied to uncertain and distant future receipts. The other is a constant which describes the "extrapolative tendency," i.e., the tendency to believe that current deviations from the average profile will be maintained in the future. The empirical evaluations, although very rough, showed that the

newly-defined variable was definitely superior to measured annual income for explaining household saving behavior. It seems likely that additional work along these lines will prove fruitful.

A third investigation focuses on Friedman's specification that permanent consumption is a constant proportion of permanent income regardless of the level of permanent income. The instrumental variable approach is used in this study with individual cities used as the classifying criteria. If average permanent income varies among cities while average transitory income is zero or a constant then it is possible to obtain unbiased estimates of the elasticity which, by Friedman's hypothesis, is unity. While the above is an oversimplification of the argument, after allowance is made for price differences and "compensating differentials" in wages, the evidence fails to provide support for Friedman's hypothesis of a unitary income elasticity of consumption.

Another area of interest was stimulated by Watts' participation in the Working Conference on Family Research Models of the Social Science Research Council. A brief paper was presented at the January 1960 meeting which outlined a model of consumer choice in which consumption of many goods and services is made to depend on a smaller number of "activity levels." It is hoped that some useful insights and hypotheses can be found by viewing consumer choice in this way.

There is little evidence in the literature of any attempt to discover the most appropriate form of curve to represent the growth of demand for a new commodity. Neither the "logistic" nor the "exponential" adjustment processes which are commonly used seem entirely satisfactory. Bain developed a model of the growth of ownership of television in the United Kingdom, in which the growth path is represented by a "cumulative lognormal" curve. The relation between the parameters of this curve and cross-section economic variables was investigated, and a time-series analysis was carried out in which the effects of changes in television service characteristics and credit restrictions on the rate of growth of ownership were estimated. The results of this analysis were free of some of the contradictions obtained in an earlier study using the logistic curve, and suggest that the "lognormal" curve might be applied more frequently in studies of growing demand.

Intergeneration Transfers of Wealth

Alfred Marshall stressed the desire to improve the welfare of the succeeding generation as the principal determinant of consumer saving.

Tobin and Guthrie undertook a pilot study of this issue. The results (CFDP 98) show that parental support of education is the most prevalent form of transfers among consumers of moderate income levels. Although public and private programs for financing the support of the older generation have grown substantially in recent years there is still a large volume of support by the younger generation.

Choice of Assets

Markowitz's study of investment portfolio selection, described in earlier reports, was published in 1959 as Cowles Foundation Monograph No. 17 under the title *Portfolio Selection* with the subtitle *Efficient Diversification of Investments.* Part III, which forms the heart of this book, discusses the simultaneous maximization of expected return, and of security of return, by suitable diversification of investment. It also analyzes how more of either of these objectives can be obtained by accepting less of the other. These analyses are preceded, in Part I, by illustrative examples of problems and answers, and in Part II, by an exposition of pertinent mathematical and statistical tools. Part IV, finally, discusses the principles of choice under uncertainty that underlie the entire study.

Monetary Theory and Policy

Tobin is approaching monetary theory as part of the theory of general equilibrium in asset markets. This equilibrium results from the individual choices and preferences that guide households, business firms, and financial institutions in managing their individual balance sheets. The theory of asset choice which Tobin employs is built both upon Markowitz's work on portfolio selection just described, and on Tobin's own initially independent but similar research over an extended period. His principal work during the period of this Report has been the preparation of a book on monetary theory, which was nearly completed at the time he took leave in January 1961 to accept appointment to the President's Council of Economic Advisers. The nature of the book is indicated by the following titles of chapters completed in preliminary draft form:

National Wealth and Individual Wealth
Properties of Assets
The Theory of Portfolio Selection
The Demand for Money
Growth and Fluctuation in a Two-Asset Economy

The Monetization of Capital
The Theory of Commercial Banking
The Monetary Mechanism
Financial Intermediaries and the Effectiveness of Monetary Controls
(The last of these chapters has been reproduced as CFDP 63.)

These chapters have been used, at Yale and elsewhere, in advanced and graduate instruction in monetary theory. A related paper, which will be adapted for inclusion in the book, is the research memorandum on "The Pure Theory of Debt Management" which Tobin prepared in 1960 for the Commission on Money and Credit.

The purpose of the book is to develop a theory describing the manner in which money markets, capital markets, and financial institutions accommodate the supplies of assets of various kinds to the demands and preferences of the individuals and business firms who own and manage wealth. The entire range of assets is considered: currency, bank deposits, government obligations, private debts, equities, durable consumers' goods, durable producers' goods. "Money" is not treated as a case apart. Rather equality between the demand for and the supply of monetary assets is considered as a part of general equilibrium in asset markets. Other assets are more or less close substitutes for "money," even if they are not generally accepted media of payments. The book tries to derive systematically the implications of this obvious fact. Interest rates, capital gains, equity yields, rents and profits from owning real property serve as the "prices" which adjust to equalize supplies and demands in asset markets. But these "prices" also affect—and in turn are affected by—current production, saving, and investment. Asset markets are closely linked to markets for goods and services. One of the principal objectives of the book is to describe these links, in order to illuminate the consequences which monetary events have for business activity and economic growth. In this respect, the book elaborates and builds upon Tobin's paper "A Dynamic Aggregative Model" (*Journal of Political Economy,* April 1955, pp. 103-115).

The key to monetary control is the government's ability to determine the supplies and sometimes the yields of certain basic assets (currency, bank reserves, and government debt). The mechanics and objectives of monetary policy and debt management are discussed from this point of view.

An econometric study of the effects of monetary policy on interest rates was made by Okun at the instigation of the Commission on Money and Credit. In a paper prepared during, but circulated after, the period of this

Report ("Monetary Policy, Debt Management and Interest Rates: A Quantitative Appraisal," CFDP 125), he presents estimates of the effects of various monetary policy and debt management operations on the yields of long-term U.S. government bonds and Treasury bills. The results are obtained from aggregative time-series data for quarterly periods covering 196 to 1959. For all standard types of policy actions the estimated effects on interest rates are in the expected direction; as anticipated, the short rate is shown to be much more volatile than the long rate.

The most striking and most controversial aspect of Okun's results is that open market operations appear to have very similar effects whether they are conducted by means of bills or of long term bonds. Therefore, swapping bills for bonds has a very slight estimated effect on yields. Theoretically, this suggests that government securities of different maturities are close substitutes to a substantial group of investors. Yet, the rate differential is variable: it is determined principally by the overall degree of tightness in financial markets. The greater volatility of the bill rate means that the differential of long yields over short yields narrows as the entire interest rate structure rises. Inelastic expectations about future interest rates can readily account for the greater variability of short rates.

Lovell studied some aspects of the theory of inflation (CFDP 90). Money and the rate of interest were reintroduced into the post-Keynesian dynamic analysis of the inflationary gap in order that the consequences of financing government expenditure by creating money might be investigated. These investigations revealed that under a variety of assumptions concerning the nature of expectations about future price levels a continued injection of new money into the economy may lead to a *forced saving equilibrium* characterized by a stable real money supply and constant rate of price increase. Conditions likely to cause the economy to degenerate into a situation of runaway inflation were also specified in this study.

Competitive Equilibrium and Games of Strategy

Debreu's fundamental study in this field, *Theory of Value, An Axiomatic Analysis of Economic Equilibrium,* was published in 1959 as Cowles Foundation Monograph No. 16. The first few chapters introduce mathematical representations of producers' and consumers' opportunities and motivations, and of the notions of commodities and prices through which these opportunities and motivations are expressed. Chapter 5 defines equilibrium and states conditions under which a private ownership economy admits of an equilibrium. Chapter 6 indicates in what sense such an equi-

librium can be called optimal, and conversely proves that any such optimum can be realized by an equilibrium. The last Chapter 7 extends these results to the case where producers' opportunities and consumers' opportunities and preferences relating to future periods are subject to uncertainty. References to earlier work of other authors are appended to each chapter.

In subsequent work, Debreu has further unified and strengthened the various results obtained on the existence of an equilibrium. This is achieved by introducing the concept of a quasi-equilibrium and proving a general existence theorem for quasi-equilibria. Simplifying somewhat, one may say that this new concept differs from an equilibrium in that consumers are treated as expenditure minimizers for a given utility level rather than as utility maximizers for a given expenditure level. By this device one can bypass one of the main difficulties met with in the study of this problem and later conclude easily from the existence of a quasi-equilibrium to that of an equilibrium.

Besides the existence and the optimality properties of competitive equilibrium, recent literature has been occupied with the problem of stability of equilibrium. Scarf, in a study (CFDP 79, A) begun at Stanford University and completed at the Cowles Foundation, contributed to this topic by constructing a few counter-examples to conjectures previously hoped to be true. These examples relate to markets in three commodities, in which the price adjustment process is described by relating the rate of change in each price to the excess demand for the corresponding commodity. The excess demand is in turn derived from demand functions obtained through utility maximization. In the examples shown, no equilibrium is approached over time from any initial non-equilibrium set of prices.

In the foregoing studies equilibrium means *competitive* equilibrium, in the sense that each producer, trader or consumer takes market prices as given to him, either because he has no influence over them or because he does not use to his own advantage such influence as he might have. This is in contrast with games of strategy where all available moves may be used. It has been harder to analyze the implications of that more realistic assumption. In studying a special class of market games first introduced by Edgeworth in 1881, Shubik proved (A) and interpreted (CFDP 107) a property which if extended to more general cases may become an important link between game theory and the theory of competitive equilibrium. The *core* of a game is defined as the set of all those outcomes (or "imputations") that cannot be changed to its own collective advantage by *any*

coalition of players within the rules of the game. For the games in question, Shubik found that if the number of participants on both sides of the market is increased in proportion without limit, the core of the game converges to the corresponding competitive equilibrium.

Another method for the study of game situations is through experimental play with a moderate number of players. Shubik (CFDP 105) made some experiments with 10 Yale seniors as the players, to study the effect of the structure of the game on behavior, and the learning processes that go on during the play. He is also constructing a more detailed business game for both teaching and research purposes (CFDP 115).

Production Capability

In economy-wide planning or forecasting, the productive capacity of an economy as of a certain year is often expressed by a Gross National Product (GNP) figure computed at the prices of some base year. While adequate for some purposes, this procedure taken literally would imply that production capability so measured is independent of the particular product-mix that is desired. However, specific capacities, bottlenecks, and substitution possibilities enter into the appraisal whether any given product-mix that stays within the total GNP estimate is actually feasible. Process Analysis, a technique for making more detailed capability estimates that depend on the product-mix, was the topic of a symposium, held April 24-26, 1961 at the Cowles Foundation, organized by Manne and Markowitz. Participants in the symposium, other than Cowles Foundation staff members, included Anne Carter, Harvard Research Project on the Structure of the American Economy; Tibor Fabian, of Lybrand, Ross Bros. and Montgomery, New York City; Daniel Gallik, New York City; Earl Heady, Center for Advanced Study in the Behavioral Sciences; Marvin Hoffenberg, Operations Research Office, Bethesda, Md.; Walter Isard, Wharton School, University of Pennsylvania; Thomas Marschak, University of California; T.Y. Shen, Wayne State University, Detroit; Thomas Vietorisz, United Nations; Marshall Wood, National Planning Association, Washington, D.C. Manne and Markowitz will also edit a forthcoming Cowles Foundation Monograph, "Studies in Process Analysis," containing the papers presented at the symposium. This collection of papers is intended to argue the desirability and demonstrate the feasibility of constructing economy-wide interindustry models on the basis of technological data, rather than of historical money flows alone. Each of the individual papers is formulated in terms of an "activity analysis" framework. The three

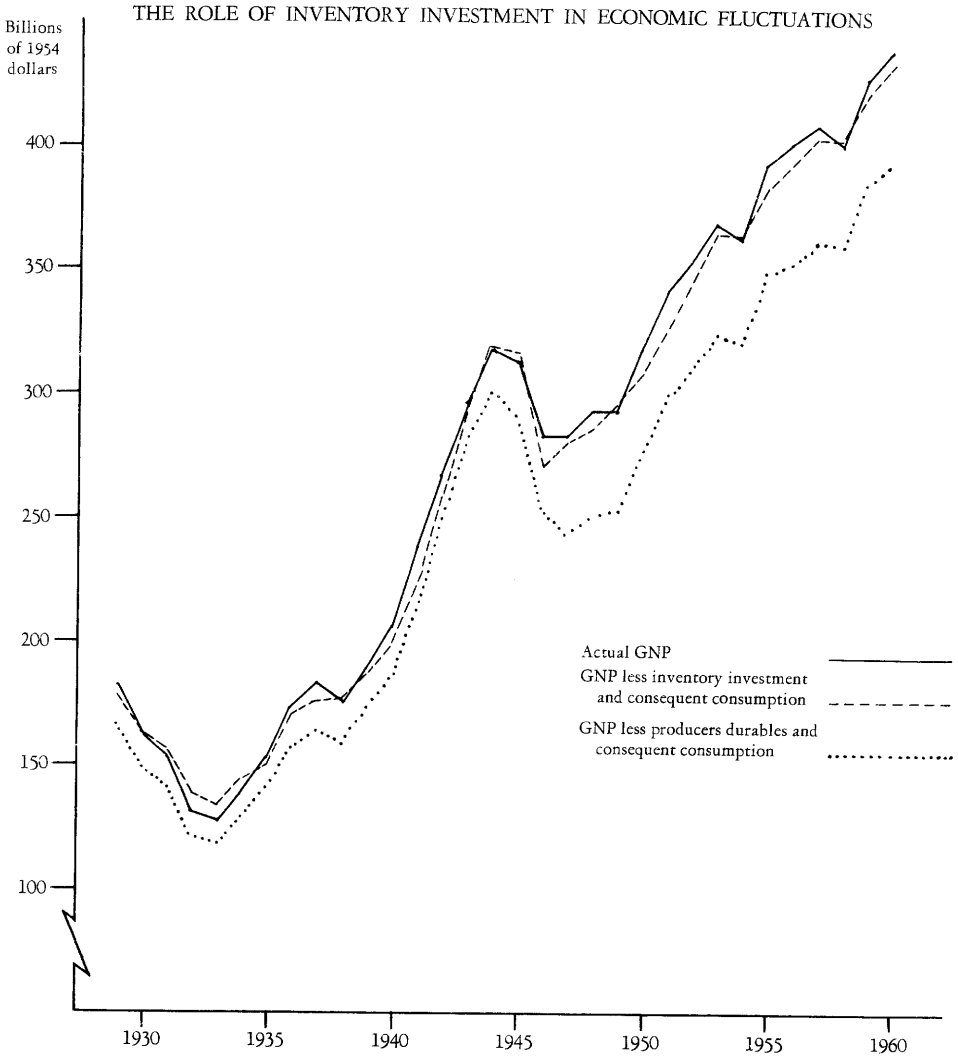
substantive sections of the volume are devoted to the following sectors: (1) the petroleum and chemical industries—manufacturing, transportation, and plant location; (2) agriculture; and (3) primary metals and metalworking. The prospective table of contents is shown on p. 48.

Inventory Fluctuations

Lovell has made a systematic study to relate inventory practices of individual firms to cyclical fluctuations in the level of aggregate economic activity. The following chart contrasts the course of Gross National Product (GNP) over the past thirty years with a hypothetical GNP series representing what gross output would have been if there had been no net inventory investment. The hypothetical series is obtained by subtracting from actual GNP both inventory investment and an estimate of the consumption it generates. The chart shows that reductions in inventory deepened the trough of the great depression of the thirties. Both the 1937 and 1949 recessions appear to be explained by declines in inventory investment. A sizable replenishment of stocks did smooth the task of reconversion following World War II. At the close of the Korean emergency, on the other hand, a reduction in inventory investment deepened the recession.

The effect upon GNP of investment in plant and equipment is revealed by a second hypothetical series plotted on the chart. (This second hypothetical series is derived under the assumption that inventories and other categories of investment were not affected. On the other hand, the full multiplier effects through consumption are taken into account. The effects of zero investment in plant and equipment upon productivity are excluded.) It appears that investment in plant and equipment generates a much larger proportion of effective demand than inventory investment. Although a considerable contribution to economic growth may be attributed to fixed investment, however, fluctuations in inventory investment must bear primary responsibility for cyclical movements in output. With the exception of the post World War II reconversion, inventory investment has been perverse in timing and magnitude, contributing to the generation of cyclical fluctuations, to booms and unemployment. An explanation of this perverse characteristic of inventory behavior requires both empirical and theoretical research.

Lovell's investigations suggest that inventory investment can be explained by an accelerator model complicated by delayed adjustment and by errors in forecasting sales volume. In an investigation of manufacturing



inventory behavior (CFDP 86) quarterly deflated time series data for five durable goods industries and the nondurable manufacturing aggregate

were utilized. A more recent investigation based on deflated quarterly GNP data suggest that the same model serves to explain the behavior of the non-farm business inventory aggregate. The evidence suggests that inventory investment is insensitive to interest rate changes. Speculative purchasing of inventories in advance of anticipated price changes does not appear to be of major importance. Errors in anticipating future sales volume are small in magnitude; while this result means that on the average firms forecast accurately, this is no doubt partly due to a cancelling of individual errors. The empirical investigations lead to the conjecture that inventory investment in the United States might have been even more perverse in timing and magnitude during the postwar period if it were not for errors of expectations and delayed adjustment behavior. This stands out clearly in the Korean war period.

A more complicated method of analysis was employed in the continued investigation of this conjecture. It was necessary to construct a theoretical model of the inventory cycle and to explore its stability properties for alternative values of the parameters of the system (CFDP 89). The approach of traditional aggregative analysis was rejected in favor of a framework of many sectors, as it was felt that interactions between individual firms are a crucial factor in explaining the generation of cyclical fluctuations in total inventories. Theoretical analysis of the stability properties of the many-sector model revealed that small marginal desired inventory coefficients, delayed adjustment behavior, and errors of expectations contribute to stability; on the other hand, it was found, within the context of this quantity adjustment model in which production takes time, that the assumption of correct if myopic anticipations implies that the economy is unstable. An investigation of certain properties of linear stochastic difference equations has helped determine the effects of random shocks upon the behavioral characteristics of the model. In principle, the theoretical model is capable of empirical implementation; in practice, only limited progress has been made thus far in assembling estimates of the various parameters of the system into a complete system of equations describing the inventory cycle.

Forecasting of Economic Activity

The cyclical character of fluctuations in national income has created particular interest in the prediction of turning-points in the direction of economic activity. In CFP 144, Okun stresses the need for an objective criterion of accuracy in evaluating and refining techniques for predicting

turning-points. During a slump, it is safe—and hence devoid of content—to assert that an upturn is coming. The statement becomes meaningful only when a date is attached to the prediction. The purpose of forecasting turning-points is to improve the estimate of their timing. The paper advances a suggested method for evaluating the accuracy of the predicted dating of business-cycle peaks and troughs.

An ideal criterion of predictive value would consider the benefits or costs due to the influence of the forecast on decision-making. But no such attempt is made in this paper. The suggested technique of appraisal—forecast-month scoring—simply counts correct and incorrect predicted changes in the direction of economic activity. The operation of forecast-month scoring is illustrated on historical data for naive forecasts, projections based on the distribution of lengths of past business-cycle phases, leading indicators, and diffusion indices of the National Bureau of Economic Research. Premature warnings of recession appear to be the most serious danger in the mechanical use of leading indicators.

Two earlier evaluations of forecasting procedures (CFDP 40, 45), described on pp. 18-20 of our previous report, were extended to include more recent experience and were published during the present report period (CFP 153, 135, respectively).

Okun also investigated relationships between output and employment in an effort to improve prediction of unemployment and to estimate the potential output of the economy. A number of statistical relationships suggest that each one-percent increment in real Gross National Product is associated with a decline of about one-third of a percentage point in the ratio of unemployment to the civilian labor force. The percentage gain in output far exceeds the reduction in the unemployment ratio because increased economic activity leads to longer hours of work, greater participation in the labor force, and higher productivity. The retarding effect of recessions on the growth of productivity is clearly in evidence.

Terms of Bank Lending

Hester completed a doctoral dissertation (Yale 1961) which investigated commercial bank lending. The terms at which a bank is willing to lend, such as the rate of interest, maturity, amount, etc., were assumed to be a function of the financial strength of a loan applicant, the portfolio position of the bank, and the "tightness" of the money market. Multiple regression analyses of samples of term loans and samples from the 1955 and 1957 Federal Reserve surveys of business loans confirmed this hypoth-

esis. Significant variables included a borrower's profits, his profit rate, his ratio of current assets to current liabilities, his demand deposit balances, and his total assets; the lending bank's deposit instability and loan-deposit ratio; and the prime loan rate of interest in the money market.

A bank may, of course, lend to firms in a similar financial position at very different terms. Using canonical correlation, estimates of the degree of substitutability among various terms of lending were made. For example, in one sample it was found that a borrower with particular financial characteristics could expect to borrow $100,000 on an unsecured basis for one month at an effective rate of 5%, for ten months at 5.43%, for eight years at 5.86%, etc. Another borrower representing a greater risk to the lender might be able to obtain on an unsecured basis $10,000 for three years at 8%, but he must pay 10.4% for a similar loan of $20,000.

The availability of credit doctrine states that as monetary authorities cause interest rates to rise, lenders simultaneously increase their credit standards, thereby reinforcing the effectiveness of monetary policy. Hester found, with one exception noted below, no empirical evidence of such credit rationing by commercial banks. Although borrowers must pay higher loan interest rates when other interest rates in the economy rise, no evidence of increased collateral requirements, scaling down of loans, or shortening of maturities was found, except in the case of term loans by large banks where maturities appear to shorten.

It was found both in this study, and in similar research by Porter, that considerable individual, random variation exists among different loans, bank officers, banks, and borrowers. Hence statistical methods that process information about large numbers of loans are essential to investigations in this field.

International Trade

Krause made an empirical investigation of the relationships between imports and other sectors of the United States economy. It is evident from the dearth of quantifiable evidence as to these relationships that commercial policy decisions have had to be made without any real knowledge or reliable estimates of their economic consequences. Moreover, the attempt to provide such estimates using aggregate relationships and time series data is subject to inherent statistical difficulties. Finally, many of the problems of real interest are of a structural rather than an aggregate nature and thus not subject to investigation in terms of aggregates. Disaggregative studies using cross section techniques over time therefore

seem more promising. Krause's first effort (A) to study the relationship between changes in our commercial policy and the United States economy was a modest attempt to appraise the effect of a particular group of tariff reductions on the level of imports. By comparing over time the growth of imports of a group of products that have been the recipients of legislated tariff reductions to a similiar group of products without such reductions, one can test for the significance of the observed differences in effects. While this work was aimed at a narrow question, it became clear that many of the assumptions concerning non-included variables were unrealistic and thus the conclusions were far from completely satisfactory. It was therefore deemed necessary to broaden the approach and take directly into account the most important economic variables for which data were available.

The next phase of the study was devoted to making statistical estimates of an explanatory equation for the quantity of imports of the United States since World War II, using as explanatory variables import prices relative to the competitive domestic price, the height of the ad valorem tariff and the amount of domestic production of the comparable product. The results of this part of the study are most encouraging and indicate that some insight has been gained as to the determinants of U.S. imports from 1947 to 1958 (CFDP 102). Within a given level of aggregate imports, mainly determined by aggregate income, relative prices are particularly important in determining the patterns of trade. This is by no means a surprising result but the indicated degree of responsiveness of imports to price changes adds weight to other evidence against the 'elasticity pessimism' school of thought. Tariff changes made after 1947, on the other hand, do not seem to have important effects. This result is somewhat unexpected in view of the political heat that the tariff issue generates. However, the large tariff reductions made in the year 1947 did lead to significantly larger imports. These seemingly conflicting results can be explained by recognizing that the observed range of variation of tariffs has been very narrow since 1947 as compared to the large change in 1947 itself, and that the procedure for choosing the products for tariff reductions has been drastically altered since 1947. While imports are likely to have replaced domestic production in some lines of activity, the results of this study indicate that in general increasing imports are associated with growing domestic production. This finding has major political importance in that adjustment to freer trade is always easiest within a growth setting.

The recently completed final phase of Krause's study (CFDP 119) is an attempt to investigate to what extent import competition affects the

pricing behavior of oligopolistic industries. A theoretical model was considered with alternative possible links between imports and oligopolistic pricing. The steel industry was selected for empirical study since it is a highly concentrated industry with many products facing various conditions of import competition. The statistical results indicate that for the period studied (1954-1958), domestic prices were unresponsive to changes in the conditions of competitive imports. This can be explained by institutional factors within the steel industry, and by the fact that imports are small relative to domestic production. While this result may not hold for other industries or even for the steel industry in some other time period, it is also clear that the discipline of import competition will not always "restrain" administratively determined domestic prices.

A study of the revenue received from foreign tourists in 58 countries in the years 1955-56 was made by Guthrie (CFDP 93) in an attempt to discover some economic explanations for differences in receipts between countries. Travel fares between countries, the volume of exports, and the amount of emigration in the past are associated with differences in receipts from tourists. After allowing for the effects of these variables in a regression equation, the residuals are interpreted as a quantitative measure of the differences in qualitative attractiveness of the various countries for tourists.

Labor Mobility in Agriculture

The tendency for agricultural income in some areas of the United States, notably the Southeast, to be markedly below incomes in the nonfarm sector has quite generally been interpreted as a symptom of immobility of the farm labor force—the counterpart in agriculture of "frictional unemployment" in the industrial sector. It is not clear, however, that the behavior of farm persons of all ages can be so simply assessed.

Berry made an empirical analysis by narrow age categories of the net migration of farm labor from agriculture during the decade 1940-1950. The analysis considered white rural farm male persons who were between the ages of fifteen and sixty-four in 1940. Rates of net migration from the rural farm labor force were obtained for each of the ten five-year age groups 15-19, 20-24, ... - . , 60-64. Explanatory variables included measures of 1940 farm income, farm income change during the 1940-1950 decade, distance to and size of the nearest major city, and the degree of farm ownership within the farm labor force Substantial differences were found in the behavior of farm persons of different ages. For the younger

age groups, the relationship between off-the-farm migration and the 1940 level of farm income was both significant and negative as expected. On the other hand, this relationship tended almost to disappear for the intermediate age groups (such as 30-35), and became noticeably positive when older workers were considered. Low farm income, which appropriately acts as an incentive to migration by younger farm persons, is evidently a deterrent to the migration of older workers, who with higher incomes would retire to nearby cities.

A high percentage of hired farm workers within the farm labor force was found generally consistent with higher than average rates of migration. The greater the distance to the nearest major city the less rapid was the migration of young farm workers, but the more rapid the migration within all other age groups, reflecting perhaps that the availability of part-time nonfarm employment may be less the more distant a major nonfarm labor market. Other findings also differ among the ten age groups.

Although necessarily preliminary, these results are suggestive. The decrease in mobility with advancing age emphasizes the long run social cost of even temporary urban unemployment which, by retarding the migration of younger persons, may more permanently commit to agriculture a significant fraction of this group.

Managerial Economics

A multiple-purpose water development project has several identifiable products, such as flood control, electric power, irrigation, etc. In such situations, it is a first problem of managerial economics to design and operate the system in such a way that no less of any of these benefits is obtained than is possible, in view of how much of each of the other benefits is obtained. Once it is known how to achieve this, a second problem is to determine quantitatively how much of any one of these benefits must be given up, if a stated increase in another one is to be obtained. Manne made a study (CFDP 95) of such "trade-off" ratios for a simplified and hypothetical project involving one storage reservoir and a three-season annual cycle of operations. This study applies a probabilistic programming technique (CFP 148) described under the heading "mathematical tools" on p. 35 of this report.

Beckmann contributed a paper on "Principles of Optimal Location for Transportation Networks" to a Symposium on Quantitative Problems in 1960. This paper is in part a further development of the "continuous Geography, organized by W. Garrison in Chicago and held on May 5-6,

model of transportation" described in the Twenty Year Report 1932-1952 (pp. 75-76) and of ideas proposed by W. Prager of Brown University. Suppose that traffic is generated with a known continuous density over an extended region and terminates at a given single point. The cost of transportation per unit distance and unit volume is constant throughout the region. One may, however, construct a trunk road passing through the point of destination on which the cost of transportation is smaller per unit distance. The capacity of the road is unlimited and the cost of construction and maintenance per unit distance is given. The problem of determining the optimal extent and location of the trunk road (more generally of a road network) may be analysed by means of the calculus of variations. The following necessary condition is obtained: At any point along the trunk road its curvature should be proportional to the excess of flow entering from the convex side over the flow entering from the concave side. Conditions for the optimal termination points of the trunk line and for the optimal angles between roads at junctions may also be found. At a junction the "forces" composed of the costs of construction and of transporting the existing flow per unit distance must be in equilibrium. (This is analogous to the well known Weberian conditions for the optimal location of a plant shipping to and/or from a set of given locations.) For a network the problem reduces to the combinatorial one of comparing the optimal graphs resulting from the various possible topological configurations of networks. This paper is to be published in the Proceedings of the Symposium mentioned above.

Another study by Beckmann is concerned with production smoothing and inventory control. Suppose that quarterly demand for a given product is a random variable, identically and independently distributed; that the cost of changing the rate of production is proportional to the absolute value of this change; and that production cost, inventory carrying cost, and shortage penalty cost are all proportional to the amounts involved. For a given inventory level and a given past production rate, what is the best new rate of production? Analysis by "dynamic programming" has shown that the optimal policy is of the following type: For each inventory level there exist two limiting rates of production; if the existing rate of production falls between these two limits, do not change it; if it is above, reduce it to the upper limit, if below raise it to the lower limit. The limiting rates of production considered as a function of inventories may be regarded as curves which border the set of points for which the production process is "in control." The shape of these curves can not be determined analytically. Subsequent computations done at Brown University for typical values

of the various cost parameters have shown that the boundary curves tend to be approximately linear with a slope of almost $-45°$.

Hooper revised and did further work on a joint paper (A) with David S. Stoller of the RAND Corporation. This paper is concerned with the problem of finding conditions under which individual service facilities should be aggregated in order to perform a certain workload in an optimal way. An example of the problem would be the following: Assume that two persons are available to service customers in a grocery store and that servicing consists of the two operations of totaling and receiving the money due from the customer and of bagging groceries. Assume further that the decision has been made that either separate parallel facilities shall be used or that the two clerks shall work as a team in one facility. The problem is, which of these two arrangements, team or separate, is optimal. This, although seemingly a simple problem, does not yield to intuitive reasoning, since the decision is shown to depend on the value of the parameters of the distributions involved as well as the efficiency of the team.

Information and Organization

The economic decision-maker is usually uncertain about the outcome of his actions. By seeking information he can diminish uncertainty and thus make his actions more effective for the achievement of his goals. By and large, information is the more useful, the more complete and accurate it is. But it is not costless. To be worthwhile, information must contribute (on the average) more towards the achievement of the goal than the cost of obtaining it. Marschak (CFP 146) asked whether and how the "value of information" to its user (its "demand price" if there is a market for information) and its cost (or "supply price") are related to the "amount of information." This latter quantity is used by communication engineers to measure the average amount of uncertainty that is removed when one of a given set of messages is received. This measure is completely determined by the number and the probabilities of possible messages, independently of their contents and their potential uses: it is the weighted average of the logarithms of those probabilities, also known to physicists as "entropy."

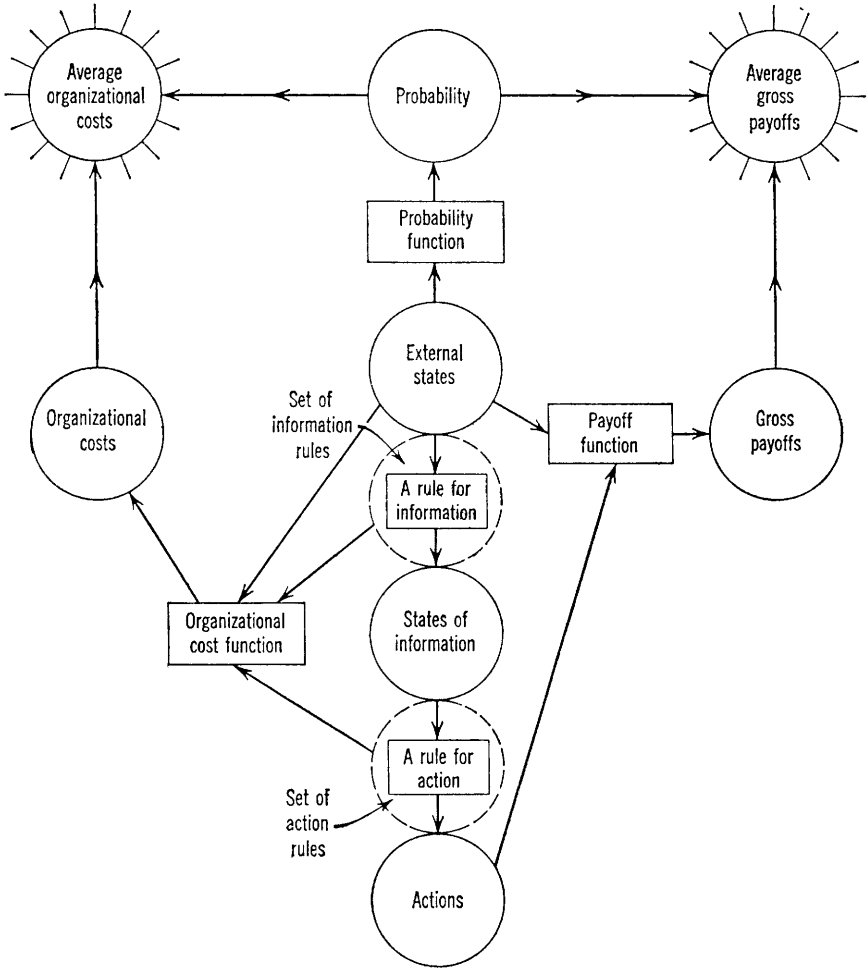
For example, all sets of three equally probable messages have the same information amount, which in turn is larger than the information amount contained in any set of two equally probable messages. Yet consider a speculator who has to choose between buying and selling a fixed amount

of stock. Suppose he can ask either of two specialists, *A* and *B,* for forecasts of the trend of next week's prices. *A* can tell him one of *two* things: whether the stock will "rise" or "not rise." *B* can tell him one of *three* things: whether the stock will "rise by 5 points or more," "fall by 5 points or more," or "change in either direction by less than 5 points." As measured by the communication engineer, *B*'s information amount is therefore larger than *A*'s if, as before, we assume in each case equal probability of the possible messages. It is also plausible that, since *B*'s kind of information is more detailed than *A*'s, *B* has to spend more effort or money to get it, and will therefore have to ask a higher "supply price" as a minimum reward that would make his sacrifice worthwhile. Yet, *A*'s forecast, though less detailed, happens to have greater value for the speculator than *B*'s. Of course, whenever the future price change is large, both *A* and *B* are of equal help for deciding whether to buy or sell stock. But, whenever the price change is moderate, *B* is of no help while *A* is. For, ignoring commissions, the speculator will buy on rise, sell on fall, moderate or not. He should consider, as a maximum offer for a specialist's services, a higher "demand price" in the case of *A* than in the case of *B.* This example illustrates the fact that the demand price, or the "value of information" to the user depends, not only on the number and the probabilities of potential messages, but also on the payoff that can be obtained by the user who wishes to choose the best decision in response to a given message.

Similar considerations apply also to the "noise" in an information channel, that is, the added uncertainty due to error. If our specialist *A* would be known to err more often than *B,* comparison between the respective "amounts of information" would be even less favorable for *A.* Yet his messages may be the more useful ones.

The subject of what constitutes useful information was further studied in McGuire's "Comparison of Information Structures" (CFDP 71). "Information structure" (or "information rule") specifies, for each state of the decision-maker's environment, the message that he will receive about the state of his environment. To use his information structure well, the decision-maker also has to fix some "action rule" (or "strategy"): this will state the best response to any given message.

The Figure on page 40a is a "flow chart," like those used in making programs for computers. In each "box" (function, "operator"), certain variables ("inputs," represented by circles) are processed into certain other variables ("outputs"). The decision-maker can choose among available information rules and action rules. But he has to accept as given: the



Circles are sets (variables)

Boxes are operators (functions)

Dotted circles ◌ are sets of ***controlled*** operators which can be chosen so as to maximize the ***net*** expected payoff, i.e., the difference between elements of radiant circles ☼

probability distribution of external events; the "payoff function," which states the usefulness of each action when performed in a given state of the environment; and the "organizational cost function," which tells how much it costs to obtain certain kinds of information, and to apply certain decision rules.

An organization of several men differs from a single decision-maker in that the goals may vary from member to member, and that in general, the members must act on the basis of different information. Marschak and Radner are continuing their work on a Cowles Foundation monograph, *Economic Theory of Teams* (projected table of contents shown on p. 49). In the theory of teams one abstracts from the divergence of goals. One may assume, for example, in a first analysis, that "perfect incentives" (e.g., well-designed bonuses) make each member act in the common interest while acting in his own. There remains the difference in information, the central feature of the theory of teams. It surely does not pay to let *all* executives of a firm, or all officers of a government agency, have identical information. But precisely what kinds of messages should a given member, entrusted with given kinds of actions, receive, either through his own observations, or from other members? What actions should he perform, and what messages should he send, on the basis of information he obtains? This will be described by the "controlled" boxes of our diagram. Which information and decision rules are best will depend on the nature of the "uncontrolled" boxes. One of these, the "organizational cost function" has so far been neglected for purposes of analysis. Instead, the values of various information structures were compared: the "gross" average payoffs obtained when a given information structure is combined with the most appropriate decision rule. Such comparisons were carried out in Radner's paper "The Evaluation of Information in Organizations" presented at the 1960 Berkeley Symposium on Mathematical Statistics and Probability (and to be incorporated in the monograph). He compared the values of information structures such as "management by exception" (reporting exceptions to headquarters, or holding emergency conferences); "partitioned communication" (with some results on the effects of the number and size of "departments"); "dissemination of independent information"; and "erroneous observations and erroneous instructions."

In each case, optimal decision rules had to be computed. It was possible to do this by differential calculus, assuming smoothness of the payoff function. This assumption also helped to measure the effect of varying certain important parameters, such as the degree of "complementarity" between the members' actions. However, the smoothness assumption was

dropped, for example, in Marschak's CFP 150 and in Radner's CFP 128. In the latter, linear programming methods were introduced.

Team problems that have arisen in practice were studied in a paper by Beckmann on airline reservations described in our previous Reports; and in a study of "Team Models of a Sales Organization" by McGuire (CFP 160) inspired by observing marketing practices in wholesale bakeries. His solution calls for methods of non-linear programming.

Only towards the end of the period covered by this report, did the group give systematic attention to the *cost* of information and decision. The "indivisibility" of each member of an organization, analogous to that of large pieces of equipment, leads to the study of "fixed" costs, and of "capacities"—not of machines but of men. Decision and information rules are associated with costs because their implementation requires efforts of problem-solving and of effective observation and communication. The ability to do these things quickly and well is limited, though varying from person to person. These limitations (neglected, incidentally, in the theory of games in its original form) cannot be assumed away without making useless any normative theory of organization,—that is, any theory of how to make an organization efficient. To assume all managers infinitely quick and wise is like assuming all industrial plants to be infinitely large.

Some experimental pilot studies on measuring "managerial capacity" in a rather narrow sense were started by Becker and Marschak, with the collaboration of Watts. Subjects had to solve problems of a simple "operations research" type, and the reward was the larger, the larger the profit that would be earned if the solution had been applied. For example: "After learning the current price, decide to sell or to postpone selling merchandise; there is no time limit on sale; storage costs are $20 per day; the price on any day can be anything between $300 and $500, with equal probabilities." The time spent on solution was measured. The solution arrived at "intuitively" by the respondent was compared with the optimal solution that can be obtained by precise mathematical reasoning of a kind that is not likely to be applied in practice under present market conditions for mathematically trained executives. A good intuitive solution should at least grasp the salient *qualitative* features of the optimal one. For example, in the case just described, the optimal solution is to sell when the price is above a certain level; this level is constant, regardless of the time elapsed. In another experiment (with no storage cost, and with limited time in which to sell) this cut-off price should, on the contrary, decrease as the deadline approaches. How much experience does a given person need to grasp the qualitative essence of solutions of such problems?

Many studies described in the present Report take their point of departure in some hypothesis or principle stating how decisions under uncertainty are made, or how it is recommended that they be made. This includes some of the models of saving, the studies of portfolio selection, of monetary theory, and some of those in managerial economics. In the present section we describe a few studies concerned with such hypotheses themselves, either through experimental test, or through theory construction facilitating such test.

One experimental study by Becker—another ramification of the concern with managerial ability—deals with consistency over time of preferences with regard to risk. The following experiment usually shows up a subject's inconsistency, except possibly after several repetitions have given him the opportunity to "learn." The experimenter finds the smallest amount that the subject would accept in exchange for lottery ticket that gives him a 50-50 chance of getting either 0 or 100 dollars. This cash value—call it x—may be, for example, $40 or even $30 if the subject strongly dislikes risk. If he likes a gamble he will evaluate the lottery ticket at more than $50. In this way, let the subject evaluate the cash values (to him) of the following lottery tickets:

Lottery Ticket	50-50 Chance of	Subject's estimate of Cash Value
No. 1	$0 or $100	$ x
No. 2	$0 or $ x	$ y
No. 3	$100 or $ x	$ z
No. 4	$ y or $ z	$ u

Clearly lottery No. 2 should be equally acceptable, for a consistent subject, as a lottery in which the chances of getting $0 or $100 are in the ratio of 3 to 1. Similarly, lottery No. 3 should be equivalent to one in which those chances are 1 to 3. It follows that lottery No. 4 should be equivalent to lottery No. 1, and that therefore the amount u should be the same as x. Few people exhibit this consistency (or, what is really the same thing, the ability to think sufficiently clearly and fast) on the first few trials. This and similar experiments by Becker suggest, however, that consistency tends to increase (the difference between x and u diminishes) as experience is being gained by the subject.

Another set of hypotheses explicitly accept a certain lack of consistency, or perhaps a taste for variability, but permit one to predict behavior in the statistical sense. These are hypotheses on the "stochastic man," a weakened, hence possibly more realistic, variant of the "economic man." On a pre-campaign cartoon in the *New Yorker,* a poll interviewer gets the answer. "I'd say I'm about 42% for Nixon, 39% for Rockefeller, 19% for Kennedy." Suppose we interpret this by predicting that the chances of the man's voting for these three candidates are as 42 to 39 to 19. Then the *strongest* of the stochastic choice hypotheses advanced so far, that of R. D. Luce, would suggest: if Rockefeller abandons the contest, the odds on our man's voting for each of the two remaining candidates will be still 42 to 19—any third alternative is irrelevant to the relative probabilities attached to any other two.

The *weakest* of the proposed stochastic choice hypotheses extends to pairwise choices only. Let p(a,b) be the probability that the subject will prefer a to b. The hypothesis says: if both p(a,b) and p(b,c) are larger than one-half, then p(a,c) is also larger than one-half ("weak stochastic transitivity").

The most important hypothesis of intermediate strength was proposed by Fechner a century ago, and has been used by psychologists ever since for the scaling of subjective "sensations" of various kinds. This hypothesis says that p(a,b), the probability that the subject will name *a* rather than *b* as the better (or the brighter, heavier, louder) object, is the larger, the larger the difference between two corresponding numbers v_a and v_b, called "sensations" (the economist is reminded of "utilities"). The existence of such numbers (and thus, indeed, the validity of the hypothesis) does not seem to have ever been tested statistically, presumably for lack of a precise mathematical model. However, such models are now becoming available. Our previous Report described, on pp. 4-5, an axiomatic study by Debreu of preference structures of this kind. It also describes another study by Debreu, similar in kind but different in content, of the construction of a cardinal utility scale if, for simplicity's sake, the alternatives from which experimental subjects choose are limited to sure prospects and to even chances as between just two sure prospects. Both studies, after some further improvements, were published in the period of this Report (CFP 125, 141). The mathematical tool of both studies is described below.

Marschak (CFP 155; and earlier, with Block, CFP 147) studied the logical relations between the several stochastic hypotheses here discussed, and other ones found in the literature. Some implausible implications of Luce's strong hypothesis were pointed out. This was also done, from an-

other point of view, by Debreu (B). Some further experimental work on choices between lotteries, or "investment portfolios," was started at the Cowles Foundation and continued in 1960-61 (by Becker, De Groot and Marschak) at the Western Management Science Institute, University of California at Los Angeles, with the support of the Behavioral Research Service of the General Electric Company. One provisional result is similar to the one obtained in the experiments on consistency: At least the stronger stochastic hypotheses do not begin to predict well till a learning period has elapsed. The subject's initial lack of consistency in his attitude toward risk is better explained by an adherence to some arbitrary,—as it were "magical"—patterns, chosen perhaps on grounds of symmetry, or apparent simplicity. Sometimes, after a learning period, and often quite suddenly, behavior becomes strictly consistent and hence predictable, rather than stochastic: "in a flash," the subject conceives what he really wants. This poses again the question: by what methods of selection and training can society increase the number of organization leaders and decision makers who are able to perceive organization goals and to grasp the essence of a given risk situation?

Statistical Tools of Econometric Research

Many econometric models posit relationships (often linear) between a set of dependent variables, a set of independent variables, and a set of unobservable random variables or "disturbances." The "parameters" of such a model, i.e., the behavior constants one wishes to study, are initially unknown and are estimated using sample observations on the dependent and independent variables. Once the parameters have been estimated the model can be used to predict the values of the dependent variables given some values for the independent variables. Econometric models are evaluated on the basis of how well they predict. The predicted values of the dependent variables will usually be different from the true values which actually occur in the prediction period. Thus an inevitable "prediction error" occurs. This error can be divided into two parts. One part is due to the fact that estimated parameters rather than the true parameters are used, and the other part is due to the random disturbance which occurs in the prediction period. While the size of this forecasting error is of greatest interest at the time the prediction is made, of course at that time only a probabilistic statement can be made about it. Hooper and Arnold Zellner have discussed (CFDP 77 R) the construction of a probabilistic forecast region for the error of forecast when several de-

pendent variables are predicted simultaneously from a multi-equation regression model.

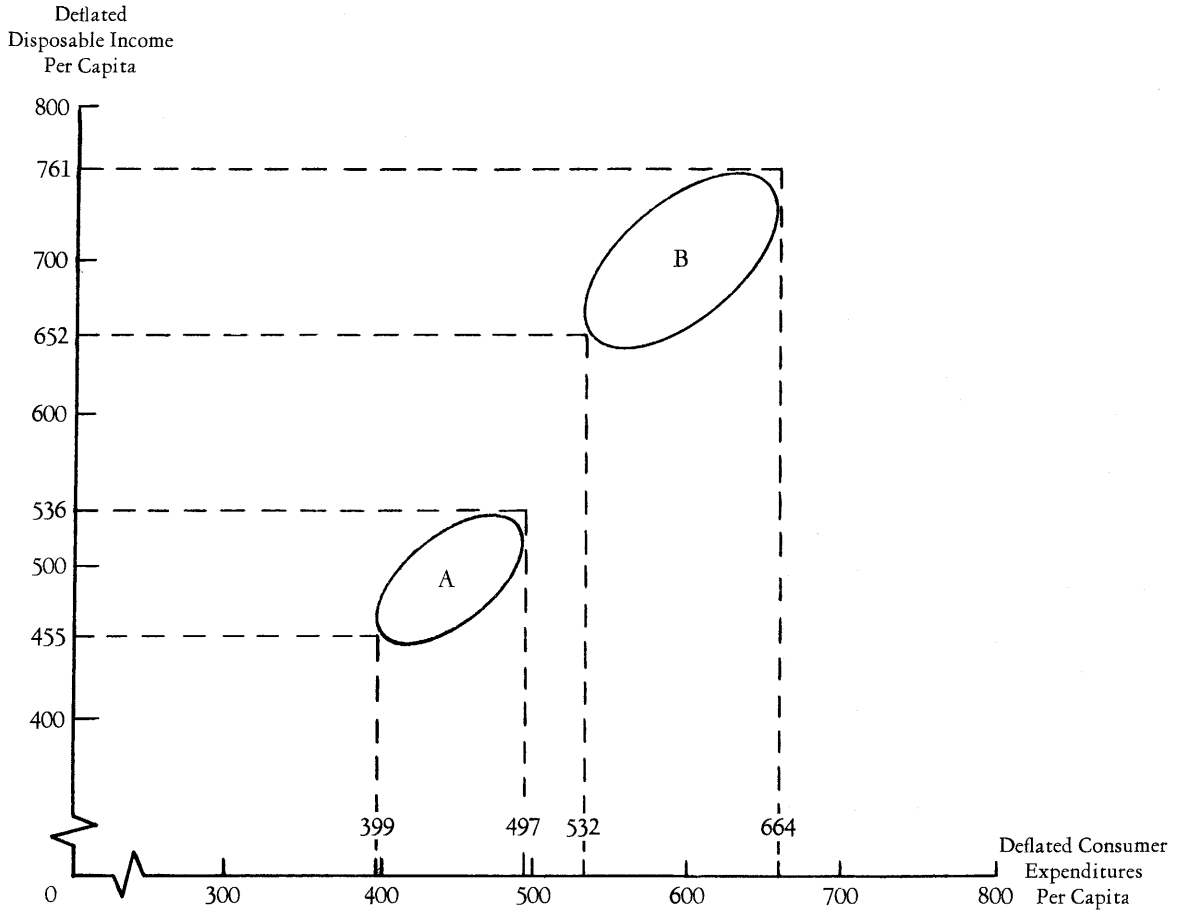
As an example of such a multivariate forecast region we can consider the following model, due to Haavelmo (*Journal of the American Statistical Association,* March 1947, pp. 105-122):

$$\begin{aligned} c_t &= \pi_1 x_{1t} + \pi_2 x_{2t} + v_{1t} \\ y_t &= \pi_{21} x_{1t} + \pi_{22} x_{2t} + v_{2t} \end{aligned} \qquad t = 1, \ldots, T$$

where the jointly dependent variables are deflated consumers' expenditures per capita (c_t) and deflated disposable income per capita (y_t). The independent variables are a constant $x_{1t} = 1$ and gross investment dollars per capita, deflated (x_{2t}). The "disturbances" are v_{1t} and v_{2t}. Two forecast regions are presented in the diagram for this model. The meaning of ellipse A is that, on the average, 95% of the time the true values of c and y will be contained in ellipse A when a prediction is made using as values for the independent variables $x_{1F} = 1$ and $x_{2F} = 100$. Ellipse B indicates a similar forecast region when the values of the independent variables in the forecast period are $x_{1F} = 1$ and $x_{2F} = 200$.

Hooper has also been concerned with the development of a criterion by which multi-equation models can be evaluated on the basis of sample observations. Assume that an investigator has two alternative multi-equation models, which are equally plausible on logical grounds, how does he decide on the basis of sample observations which model is to be retained for further use? For single equation models one criterion which is used, among others, for a choice between models is that one prefers the model which has the larger multiple correlation coefficient R^2, as computed from the sample. Analogously, in a study (B) completed before joining the staff, Hooper developed a generalized correlation coefficient for multi-equation models, called the trace correlation. A logical further extension of the trace correlation is the concept of generalized partial correlation coefficients. For a single equation, partial correlation coefficients have been useful in indicating which independent variables should be used in a model. Analogously, Hooper has developed generalized partial trace correlation coefficients for multi-equation models (CFDP 97). These partial trace correlation coefficients give a measure of how much of the variation in the set of jointly dependent variables is explained by a particular exogenous variable, after the influence of the other exogenous variables has been removed.

Hooper is presently finishing a study of specification errors in multi-



JOINT FORECAST REGIONS FOR CONSUMPTION AND INCOME AT A 5% LEVEL OF SIGNIFICANCE

equation models. His main result is that the trace correlation can be used to distinguish, on the basis of sample observations, between correctly and incorrectly specified multi-equation models.

Both Hooper and Watts are turning to the methodological problems of econometric studies based on "cross-section" data, obtained at one time or at successive moments of time. Watts has been enabled, through a Social Science Research Council Fellowship, to devote the year 1961-62 full-time to a systematic study of this topic. Hooper is pursuing in particular the connections between cross-section studies and the aggregation problem.

Summers continued his investigation of the small-sample properties of simultaneous equations estimators by the use of Monte Carlo techniques described on pp. 16-17 of our previous Report. Some tentative results from this study were presented in CFDP 64.

Dhrymes, in CFDP 122, shows why the least-squares estimators of the parameters of a Cobb-Douglas production function are biased estimators. He then derives a set of estimators for these parameters which are unbiased, sufficient, and consistent.

Mathematical Tools

Almost nothing is known on a theoretical level about the efficiency of the simplex procedure for linear programming problems. Experience with such problems indicates that the number of iterations required for the simplex method to converge is unexpectedly small. On the other hand, the known theoretical bounds on that number are very much higher. Scarf explored this problem and obtained small bounds for a quite restricted class of programming problems. The type of analysis required in this problem is of a subtle mathematical character, involving the relationships among the signs of the subdeterminants of a given matrix. He intends to return to the problem in the future, not only because of its inherent interest, but also because of the light it may shed on integer programming problems.

Beckmann has considered a calculus of variations problem (A) arising when a commodity is to be allocated over time subject to upper and lower bounds on both its stock and its flow. Necessary and sufficient "efficiency" conditions are developed for a piecewise continuous flow function to represent an optimal allocation. This model applies to the problem of optimal water storage in a hydroelectric system, and to various problems of production smoothing treated in the recent literature.

Manne has indicated (CFP 148) how the methods of linear programming can be used to solve sequential decision problems in probabilistic

models usually studied by the method of "dynamic programming." The essential idea is to adopt, as the unknowns of the problem, not the decision rule sought, but the discrete joint probability distribution of the state of the system and of the decision adopted, assuming that that decision rule is followed. The assumption of an infinite horizon serves to make that distribution independent of time. Once it is found by linear programming methods, the decision rule sought is easily obtained. One application to a multiple-purpose water development project was discussed above. Another one lies in the field of inventory control under probabilistic demand conditions.

The two studies by Debreu on the axiomatics of cardinal utility, already discussed, both employed as their principal mathematical tool a theorem of Thomsen and Blaschke that gives a topological characterization of three families of parallel straight lines in a plane. The same tool can also be applied to another problem in utility theory that was previously treated by calculus techniques. The problem is that of independent commodities, i.e., of finding sufficient conditions for the preferences of a consumer for n-commodity bundles to be representable by a sum of n functions

$$u_1(x_1) + \cdots + u_n(x_n)$$

where x_i denotes the quantity of the i^{th} commodity. Debreu showed (CFP 156) that, aside from continuity conditions, it is not only necessary, but also sufficient, for the preferences to satisfy the following independence condition: Fix any m of the x_i and consider the resulting preferences for the bundles of n - m remaining commodities whose quantities can still vary. Then these preferences should be independent of the particular values chosen for the x_i that have been fixed, and this should be so regardless of how many and which variables were fixed. Since the method used is independent of differentiability assumptions, the result is more natural, and can be readily generalized to independent groups of commodities rather than single commodities.

In their work on the theory of stochastic choice, Block and Marschak had been led to establish a certain identity arising from probability considerations and tying together different aspects of stochastic behavior. Debreu gave a brief proof of that identity based on a probability-theoretical argument (CFP 149).

Our Report for 1952-54 mentions on p. 24 a paper in which Debreu has gathered several of the separation theorems for convex sets most useful in economic theory, with proofs. This note has now been published as an appendix in CFP 136.

RESEARCH CONSULTANTS

A Research Consultant to the Cowles Foundation is a scholar at some other institution who maintains an active interest in the research program of the Foundation, manifested in exchanges of ideas and results with members of the Foundation's staff. Some Consultants are previous members of the staff, and some are completing research begun at the Cowles Commission or Foundation or pursuing further investigations stimulated by such research. Where a real relationship exists between the work of a Consultant and the program of the Cowles Foundation, the Foundation welcomes the opportunity to include the results in its publications.

The following were Research Consultants during the whole or part of the period covered by this report.

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GUESTS AND VISITS

The Cowles Foundation is pleased to have as guests advanced students and scholars from other research centers in this country and abroad. Their presence both stimulates the work of the staff and aids in spreading the results of its research. To the extent that its resources permit, the Foundation has accorded office, library, and other research facilities to guests who are in residence for an extended period. The following were associated with the organization in this manner during the past three years.

RAGNAR BENTZEL (Sweden). October 1958. Sponsored by Industriens. Utredningsinstitut, Stockholm, Sweden.

ROBERT EISNER (U.S.A.). August-December 1958. Sponsored by the Ford Foundation. Returned to Northwestern University, Illinois.

WALTER D. FISHER (U.S.A.). September 1960-August 1961. Sponsored by the John Simon Guggenheim Memorial Foundation. Returned to Kansas State University.

HOLGER GAD (Denmark). September 1958-January 1959. Sponsored by the Rockefeller Foundation. Returned to University of Aarhus, Denmark.

BERNARD GOODMAN (U.S.A.). September 1958-August 1959. Sponsored by the Ford Foundation. Returned to Wayne State University, Michigan.

GEORGE G. JUDGE (U.S.A.). September 1958-June 1959. Sponsored by the Social Science Research Council. Returned to Oklahoma State University.

JOZEF LUKASZEWICZ (Poland). January-June 1960. Sponsored by the Ford Foundation (through the Institute of International Education). Returned to Mathematical Institute of the Polish Academy of Sciences, Wroclaw, Poland.

EDWIN S. MILLS (U.S.A.). February-August 1961. Sponsored by the Ford Foundation. Returned to Johns Hopkins University, Maryland.

JEAN GERARD MORREAU (Netherlands). July-October 1959. Sponsored by the Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek. Returned to Amsterdam.

JOHN DAVID PITCHFORD (Australia). August-December 1959. Sponsored by the Rockefeller Foundation. Returned to the University of New South Wales, Australia.

D. V. RAJALAKSMAN (India). October 1959-May 1960. Sponsored by the Rockefeller Foundation. Returned to University of Madras, India.

GEORGE J. STOLNITZ (U.S.A.). September 1959-September 1960. Sponsored by the National Science Foundation. Returned to Indiana University.

BJORN THALBERG (Norway). September 1959-July 1960. Sponsored by the Rockefeller Foundation. Returned to University of Oslo, Norway.

ARNOLD ZELLNER (U.S.A.). January-September 1959. Sponsored by the National Science Foundation. Returned to University of Washington.

During the year 1958-59 the Director, James Tobin, was in Europe on sabbatical leave. He used the opportunity to have a useful exchange of views and experience at major centers of econometric research in Europe: Rotterdam, Econometric Institute and Netherlands Economic Institute; London School of Economics; Oxford Institute of Statistics; Department of Applied Economics, University of Cambridge; University of Manchester; IFO-Institute für Wirtschaftsforschung, Munich; University of Stockholm, and Konjunkturinstitutet, Stockholm; University of Uppsala; University of Oslo.

COWLES FOUNDATION SEMINARS

July 1, 1958—June 30, 1961

1958

October 10. ROBERT EISNER, Northwestern University, "Capital Expenditures and Expectations."

October 14. RAGNAR BENTZEL, University of Uppsala, "An Investigation of the Swedish Consumption Patterns."

November 7. ERICH SCHNEIDER, University of Kiel, "On the Influence of Changing Exchange Rates on the Balance of Payments."

December 12. LAWRENCE R. KLEIN, University of Pennsylvania, "An Econometric Model of the United Kingdom—Postwar Quarters."

1959

January 9. DANIEL ELLSBERG, Harvard University, "The Theory and Practice of Blackmail."

January 23. GERALD THOMPSON, Ohio Wesleyan University, "A Further Generalization of the von Neumann Dynamic Model."

February 13. ROBERT STROTZ, Massachusetts Institute of Technology, (on leave from Northwestern University) "Price Expectations, Optimality, and Equilibrium."

March 13. H. S. HOUTHAKKER, Stanford and Harvard Universities, "International Comparisons of Consumers' Preferences."

March 26. MARCEL BOITEUX, Electricité de France, "La Tarification Marginaliste de l'Electricité de France."

April 10. CARL KAYSEN, Harvard University, "Some New Data on Plants and Firms."

April 17. ALAIN ENTHOVEN, RAND Corporation, "The Neoclassical Theory of Money and Economic Growth."

April 24. MANFRED KOCHEN, IBM Research Center, "Some Problems in Organizational Structure."

May 25. MAURICE ALLAIS, Institute of Statistics, University of Paris, "Influence of the Capital-Output Ratio on Real National Income."

October 30. MICHAEL FARRELL, University of Cambridge and Carnegie Institute of Technology, "Some Remarks on the British Capital Market."

November 13. JAMES DURBIN, London School of Economics, "Estimation of Parameters in Time Series Regression Models."

December 4. ROBERT SOLOW, Massachusetts Institute of Technology, "Estimation of Distributed Lags."

December 11. BENOIT MANDELBROT, IBM Research Center, "A New Family of Stochastic Models of Income Distribution: The Pareto-Levy Random Variables and Processes."

1960

January 8. RICHARD R. NELSON, RAND Corporation, "Uncertainty, Learning, and Research and Development Decision-Making."

January 22. JOHN LINTNER, Harvard University, "Research on Earnings, Dividends, and Stock Prices."

February 12. ZVI GRILICHES, National Bureau of Economic Research, Inc., "Is Aggregation Necessarily Bad?"

April 8. FRANCIS M. BATOR, Massachusetts Institute of Technology, "On 'Balanced' Growth."

April 15. WILLIAM C. HOOD, University of Toronto, "Problems in the Regulation of Privately Owned Public Utilities."

April 29. JOHN DENIS SARGAN, Leeds University and University of Chicago, "Towards a More Realistic Theory of Stability."

May 13. HENRI THEIL, Director of the ECONOMETRIC INSTITUTE, Netherlands School of Economics, and Harvard University, "The Design of Socially Optimal Decisions."

October 19. GUY H. ORCUTT, University of Wisconsin, "Simulation of Social Systems."

December 16. ROBERT SUMMERS, University of Pennsylvania, "An Econometric Look at Military Cost Estimates."

1961

January 13. EDWARD B. BERMAN, Operations Evaluation Group, Navy Department, "The Normative Interest Rate."

March 10. GEORGE J. FEENEY, General Electric Company, "Oligopolistic Behavior in a Markovian Market."

March 17. RICHARD ROSETT, University of Rochester, "Models of the Stock Options Market."

March 24. MARTIN J. BECKMANN, Brown University, "Wicksell's Cumulative Process and Some Models of Economic Growth."

April 14. HENDRIK S. HOUTHAKKER, Harvard University, "Short-term Price Movements as a Stochastic Process."

April 21. WASSILY LEONTIEF, Harvard University, "Welfare Analysis as Applied to Public Enterprise."

April 28. EDWIN MANSFIELD, Carnegie Institute of Technology, "Acceptance of Technological Change."

COWLES FOUNDATION MANAGEMENT SEMINARS

July 1, 1958—June 30, 1961

These seminars, initiated in 1956, are aimed at promoting knowledge in the management sciences. The meetings serve as a medium for the two-way exchange of ideas between members of the Yale academic community and management people in Connecticut industries.

1958

Feburary 5. MARTIN SHUBIK, General Electric Company, "Maximization Aims in Business Enterprises."

April 22. GEORGE B. DANTZIG, RAND Corporation, "Linear Programming."

May 28. JACOB MARSCHAK, Yale University, "The Theory of Organization."

October 21. W. REED SMITH, U. S. Rubber Company, "Applications of Experimental Design."

November 10. ERICH SCHNEIDER, University of Kiel, "On the Realism of Marginalist Thinking in Business Problems."

November 25. RALPH GOMORY and E. M. L. BEALE, Princeton University, "Integer Solutions to Linear Programs."

1959

January 28. JULIUS ARONOFSKY, Socony Mobil Company, "Linear Programming Applications in an Integrated Oil Company."

March 11. ROBERT FETTER, Yale University, "Production Planning for a Multi-Product Facility."

May 6. WILLIAM S. STAPAKIS and KENNETH R. BLAKE, United Aircraft Corporation, "Some Theoretical Results on the Job Shop Scheduling Problem."

October 20. HARRY MARKOWITZ, General Electric Company, "Computer Simulation of Production Processes."

November 12. ARTHUR YASPAN, Lybrand, Ross Bros., and Montgomery, "Inventory Policies."

November 30. GEORGE FEENEY, General Electric Company, "Operational Games as Marketing Experiments."

1960

January 14. JOHN GESSFORD, International Paper Company, "Some Inventory Models and Their Optimal Policies."

LIBRARY OF THE COWLES FOUNDATION

MERLE E. HOCHMAN, *Librarian*

The principal goal of the Cowles Foundation library is to make readily accessible to staff members important past and current literature in economics, especially quantitative economics, and related works in mathematics and statistics. The library also accommodates other members of the Department of Economics and graduate students in their research and study programs.

The library collection includes some 3,860 books, 150 journals, thousands of pamphlets, and much recent unpublished material. About 890 of the books were acquired during the three-year period covered by this report. These can be divided by subject into the following categories: economics, 64%; collections of statistical data, 10%; statistical theory, 8%; mathematics, 7%; reference books, 4%; all others, 7%. Current books, ordered shortly after their publication, accounted for 88% of the new acquisitions.

Books circulate for a period of one month and journals for two days. They may be renewed by staff members only. Some 250 books which are in demand for graduate economics courses are kept on reserve, circulating overnight and weekends only.

THE ECONOMETRIC SOCIETY

The Econometric Society is an international society for the advancement of economic theory in its relation to statistics and mathematics. Its main object is the promotion of studies directed toward unification of the theoretical quantitative and the empirical quantitative approaches to economic problems and penetrated by the kind of constructive and rigorous thinking that has come to dominate the natural sciences. Any activity which promises ultimately to further such a unification of theoretical and factual studies in economics is considered to be within the sphere of interest of the Society.

At the present time the Econometric Society publishes a quarterly journal, *Econometrica.* It holds one European and one or two North American meetings each year. As an international organization, the officers of the Econometric Society represent many different countries. The major governing body of the Society is its Fellows. At the present time these number 126, and a maximum of six additional fellows are elected each year. Membership in the Society is open to anyone seriously interested in the objectives of the Society. Institutional memberships are also available in order to solicit the support of interested business firms and research organizations. In addition to the 1,530 members, there are 1,732 non-member subscribers to the journal, mainly libraries, business firms, and research organizations.

Three individuals, Irving Fisher, Professor of Economics at Yale, Ragnar Frisch, Professor of Economics at the University of Oslo, and Charles Roos, a research fellow at Princeton, were instrumental in the founding of the Society in 1930, two years prior to the establishment of the Cowles Commission. Initially the Society had less than 200 members, and its activities were restricted to the arrangement of small meetings at which papers were read and discussed. Because of the small membership and the minimal dues, it was not possible to publish a journal. With the founding of the Cowles Commission in 1932, a proposal was made that the Commission support the activities of the Econometric Society, and enable it, among other things, to publish a journal. After due consideration this proposal was adopted, and the first issue of the journal *Econometrica* was published in 1933. In the following years the Society grew, and with the increase in membership and subscriptions it became somewhat more self-supporting. But costs were also rising, and the Cowles Commission continued to bear a considerable portion of the administrative expenses of the Society. The two organizations were administered jointly.

With the establishment of the Cowles Foundation at Yale University, it was decided to separate the administrative functions of the Econometric Society from those of the Cowles Foundation, and if possible to draw the financial support of the Society more fully from its membership than had been done to date. A gradual reduction in the financial contribution of the Cowles Commission, begun while the Society was still located in Chicago, has been continued. At present the Society receives a contribution of $2,000 a year from the Cowles Foundation; and it is expected that this level will be maintained in the future. In 1960 the Cowles Foundation gave an additional $2,000 to the Society to help cover the cost of publishing a special issue of *Econometrica* in honor of Ragnar Frisch in the year of his sixty-fifth birthday. Efforts are being made to replace the reduction in the Cowles Foundation contribution from such sources as institutional memberships and an increase in individual memberships.

RICHARD RUGGLES
Professor of Economics, Yale University
Secretary

MONOGRAPHS

1934-1961*

The monographs of the Cowles Commission (Nos. 1-15) and Cowles Foundation (Nos. 16-17) are listed below:

No. 1. *Dynamic Economics,* by CHARLES F. ROOS. 1934. Evanston, Ill.: Principia Press, 275 pages. (Out of print.)

No. 2. *NRA Economic Planning,* by CHARLES F. ROOS. 1937. Evanston, Ill.: Principia Press. 596 pages. (Out of print.)

No. 3. *Common-Stock Indexes,* by ALFRED COWLES and ASSOCIATES. Second Edition, 1939. Evanston, Ill.: Principia Press. 499 pages. Price $6.00. New monthly indexes of stock prices, stock prices adjusted for reinvestment of cash dividends, and yield expectations; and annual indexes of yields, divided payments, earnings-price ratios, and earnings for 69 industry groups, 1871-1938.

No. 4. *Silver Money,* by DICKSON H. LEAVENS. 1939. Evanston, Ill.: Principia Press. 439 pages. A sketch of the history of the monetary use of silver, followed by more detailed consideration of recent developments. (Out of print.)

No. 5. *The Variate Difference Method,* by GERHARD TINTER. 1940. Evanston, Ill.: Principia Press. 175 pages. The history and use of this method for the analysis of time series, with new devices of treatment and extensive tables to aid calculations. (Out of print.)

No. 6. *The Analysis of Economic Time Series,* by HAROLD T. DAVIS. 1941. Evanston, Ill.: Principia Press. 620 pages. The historical development of the subject is reviewed, methods are described, and applications made to economic phenomena. (Out of print.)

No. 7. *General-Equilibrium Theory in International Trade,* by JACOB L. MOSAK. 1944. Evanston, Ill.: Principia Press. 187 pages. The modern theory of economic equilibrium (as stated by J. R. Hicks and others) applied to an important field. (Out of print.)

No. 8. *Price Flexibility and Employment,* by OSCAR LANGE. 1944. Evanston, Ill.: Principia Press. 114 pages. Price $2.75. A clarification of important concepts that have had much currency in the practical discussion of depressions and wars but remained too vague to allow useful treatment.

No. 9. *Price Control and Business,* by GEORGE KATONA. 1945. Evanston, Ill.: Principia Press. 246 pages. A study of the working of price control based on field studies among producers and distributors of consumers' goods in the Chicago area, 1942-1944. (Out of print.)

No. 10. *Statistical Inference in Dynamic Economic Models,* edited by TJALLING C. KOOPMANS, with Introduction by JACOB MARSCHAK. 1950. New York: John Wiley and Sons. 438 pages. Price $7.50. Original contributions from many authors concerning statistical problems encountered in economic model construction.

No. 11. *Economic Fluctuations in the United States, 1921-1941,* by LAWRENCE R. KLEIN. 1950. New York: John Wiley and Sons. 174 pages. Price $4.50. The methodology of econometric model construction is applied to business cycle analysis with possible implications for prediction and policy making.

No. 12. *Social Choice and Individual Values,* by KENNETH J. ARROW. 1951.

*Orders for Monographs 3 and 8 should be sent to Principia Press of Trinity University, 715 Stadium Drive, San Antonio, Texas. Orders for subsequent monographs should be sent to John Wiley and Sons, 440 Fourth Avenue, New York City. Prices are subject to change.

New York: John Wiley and Sons. 99 pages. Price $3.25. Methods of symbolic logic are applied to the question whether a social valuation of alternatives can be consistently derived from given, partly conflicting, individual valuations.

No. 13. *Activity Analysis of Production and Allocation,* edited by TJALLING C. KOOPMANS. 1951. New York: John Wiley and Sons. 404 pages. Price $5.50. Contributions from economists and mathematicians on the theory and techniques of efficient allocation of resources and programming of activities.

No. 14. *Studies in Econometric Method,* by COWLES COMMISSION RESEARCH STAFF, edited by WILLIAM C. HOOD and T. C. KOOPMANS. 1953. New York: John Wiley and Sons. 324 pages. Price $6.00. Presents and extends methods developed in Monograph 10 in an expository style addressed primarily to the user of methodology.

No. 15. *A Statistical Study of Livestock Production and Marketing,* by CLIFFORD HILDRETH and F. G. JARRETT. 1955. New York: John Wiley and Sons. 156 pages. Price $5.50. Economic relations underlying the operation of livestock markets in the United States are estimated and tested by several alternative procedures.

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Two further forthcoming monographs are in preparation, *Studies in Process Analysis: Economy-Wide Production Capabilities* edited by Alan S. Manne and Harry M. Markowitz, and *Economic Theory of Teams* by Jacob Marschak and Roy Radner. The tentative outlines of contents are as follows:

STUDIES IN PROCESS ANALYSIS: Economy-Wide Production Capabilities
(Outline of Contents)

Foreword		T. C. KOOPMANS
PART I.	*Scope and Method*	
Ch. 1	Analysis of Production Capabilities	A.S. MANNE & H.M. MARKOWITZ
Ch. 2	Alternative Methods of Analysis	A.S. MANNE & H.M. MARKOWITZ
Ch. 3	Implementation of Process Analysis	A.S. MANNE & H.M. MARKOWITZ
PART II.	*Petroleum and Chemicals: Manufacturing, Transportation, and Plant Location*	
Ch. 4	A Linear Programming Model of the U.S. Petroleum Refining Industry	A.S. MANNE
Ch. 5	A Spatial Model of U.S. Petroleum Refining	T.A. MARSCHAK
Ch. 6	Plant Location Under Economies-of-Scale	T. VIETORISZ & A.S. MANNE
PART III.	*Agriculture*	
Ch. 7	Spatial Programming Models to Specify Surplus Grain Producing Areas	E.O. HEADY & A.C. EGBERT
Ch. 8	Spatial Price Equilibrium in the Food and Agricultural Sector	KARL A. FOX

ECONOMIC THEORY OF TEAMS

(Outline of Contents)

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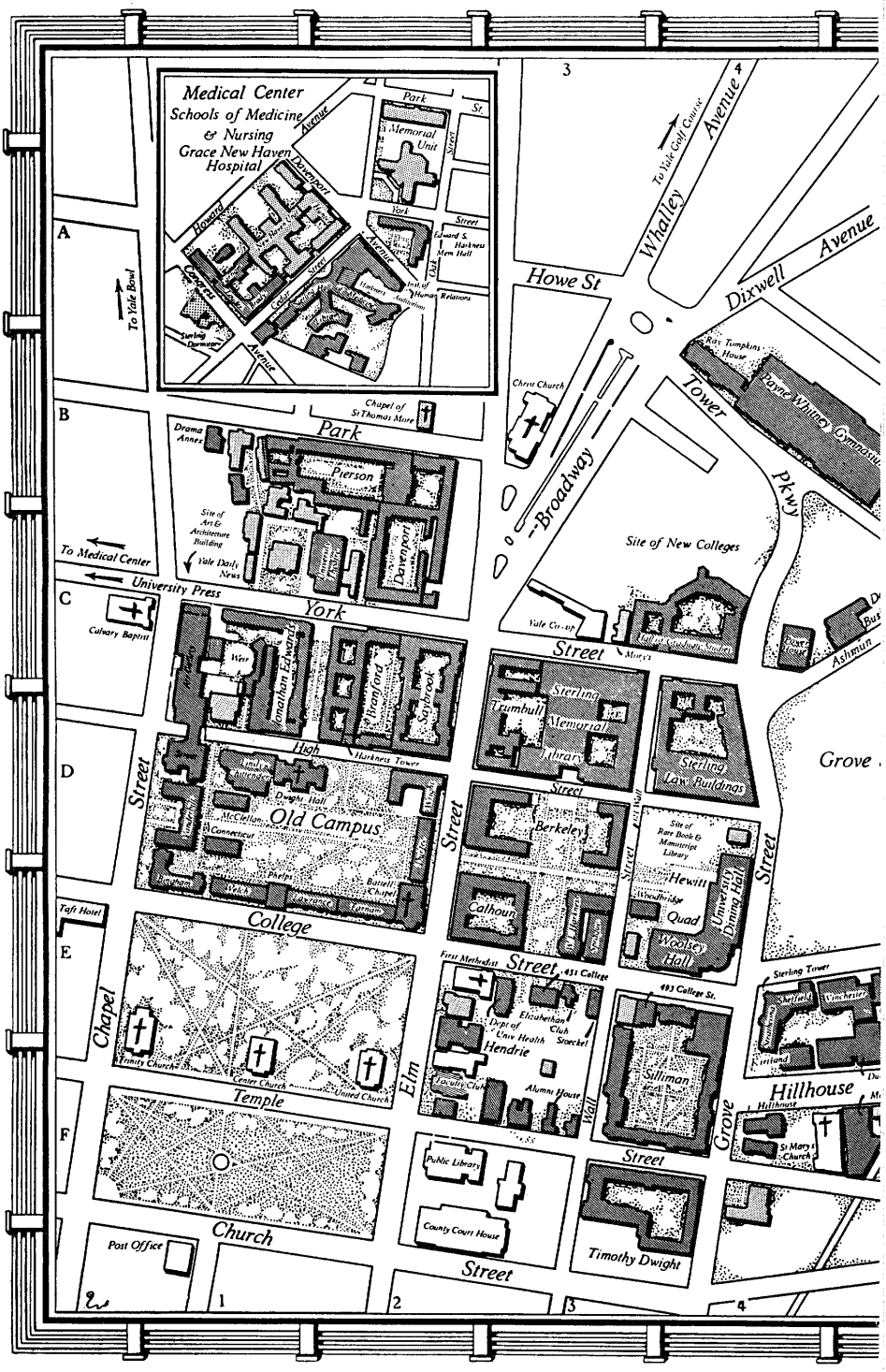
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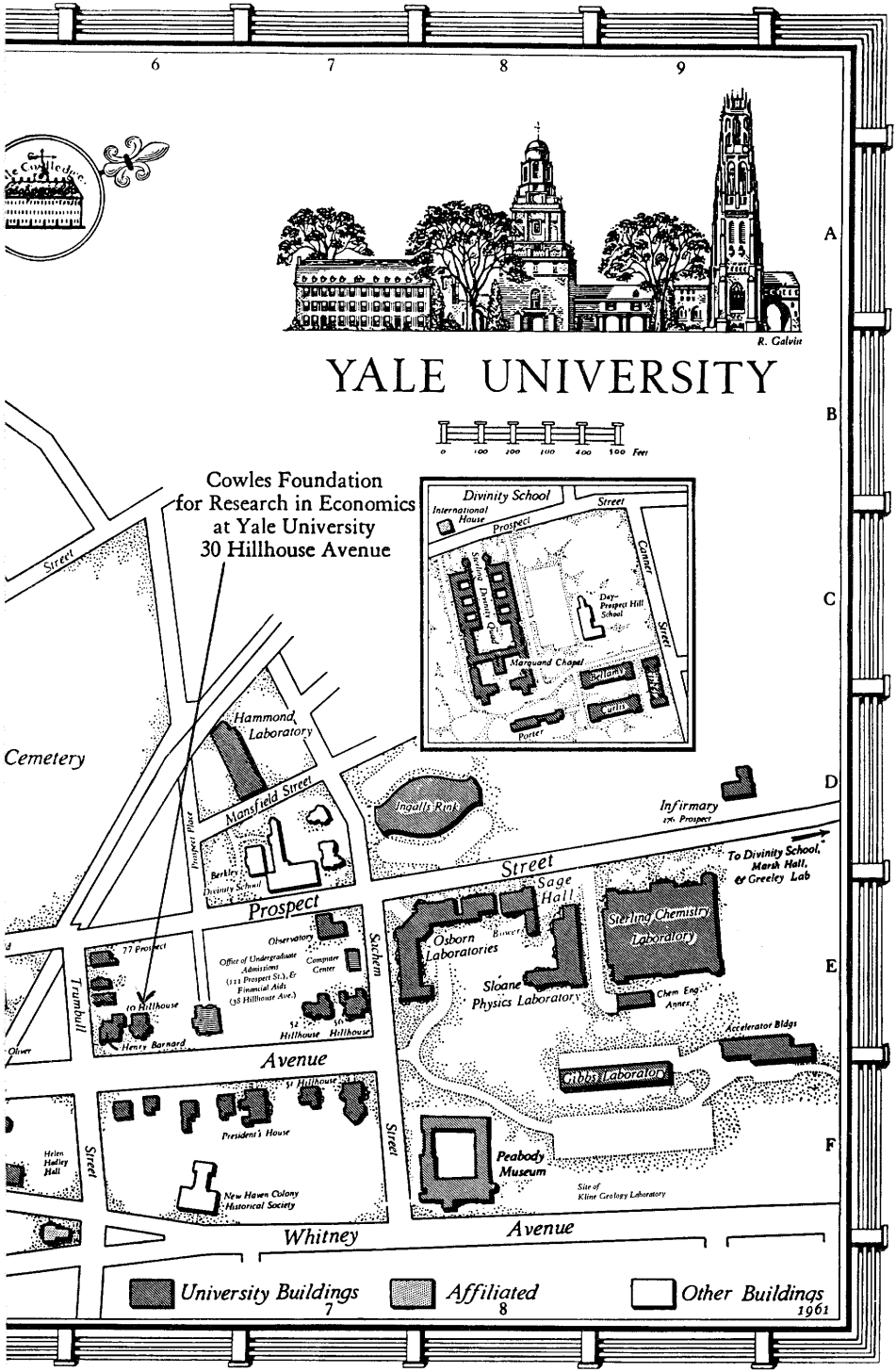
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EDWIN S. MILLS
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