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SIMULATION OF SOCIO-ECONOMIC SYSTEMS

PART II: AN AGGREGATIVE SOCIO-ECONOMIC SIMULATION  
OF A LATIN AMERICAN COUNTRY

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(revised)

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An Aggregative Socio-Economic Simulation \*  
of a Latin American Country

Martin Shubik      \*\*      \*\*\*

The simple model constructed here is built to serve as an illustration of the uses of simulation and as a first approximation for a socio-economic model to be used in connection with the investigation of development problems in parts of Latin America.

The technique of simulation serves as an excellent data organizing device and framework for the conceptual scheme behind a national economic accounts system and aggregate financial reporting system. This simulation utilizes in part the national economic accounts format developed in the Country Study Program of the Yale Economic Growth Center <sup>1/</sup> and the financial statistical reporting of the International Monetary Fund and World Bank. <sup>2/</sup>

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Nonr - 3055 (01)

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Although this first model is based primarily upon an aggregated national income system of a variety that is well known to economic studies, it contains several parameters and relations of a socio-economic variety, which are not commonly found in national income models.

The simulation described below has been constructed in a manner that employs what can at best be called "casual empiricism." It has been used to provide a structure for joint discussion and as a possible basis for research between anthropologists, economists, political scientists, sociologists, and others. The eventual goal however is to demonstrate that this type of simulation is a natural integrating device to provide a basis for discourse among operating planners, those concerned with the gathering of information or the administration of data gathering and information systems as well as behavioral scientists.

Part of the motivation behind this work is to show that relatively interesting and thought-provoking models can be constructed and examined quickly and relatively cheaply by means of computer simulation. Before validation problems must be faced, pre-validation problems must be overcome. These involve the sorting out of variables, the organization of common sense, uncommon sense, insights and "guesses," and the "playing" with models as a preliminary to well-defining structural relationships and devising methods for empirical verification of the chosen relationships.

Accuracy and validity should depend upon the question to be answered; models and questions which appear to be vague to the econometrician may thus have a place and use to the policy maker, political scientist, administrators, and others. These comments should not be interpreted as a defense of sloppy thinking. The gaps separating policy oriented studies, economic theory and econometrics as well as highly relevant sociological and political studies are large. The work here is addressed to providing an interlinkage for the different approaches.

#### 1.1 Sociological and Political Factors

This simulation is directed towards portraying some of the basic features of a Latin American country with a large Indian population. Peru, Ecuador, Bolivia, Honduras, Guatemala and Nicaragua fall into this category. In particular, as is indicated in Part III, we concentrate on Ecuador.

Models of different countries undoubtedly have much in common; however, it is our belief that we are a long way from the "all-purpose" general simulator. A common methodology, data gathering procedures and common output formats with which to carry out comparative studies can be achieved. However, in detail models will differ from country to country.

Eventually it will be highly desirable to deal directly with such factors as degree and speed of urbanization, the effects

of religious beliefs and political structure <sup>3/</sup> At this point neither time nor resources permit including these in the rudimentary model constructed here. Nevertheless the point that is fundamental to the approach adopted here is that in the study of socio-economic and political systems the best disaggregation may be a mixed disaggregation. For many purposes the choice between a disaggregation into say 40 economic sectors or 10 economic sectors and 4 social sectors should be made in favor of the mixed economic and social disaggregation.

Formal data often follow theories and purpose. Thus, the argument that the data do not exist or are hard to quantify is not a sufficient reason to abandon factors which may serve to provide much more explanation of processes of growth or change than do available easily quantifiable variables. A poor guess as to the effect of an important but hard to quantify variable may easily be of more use than an elegant time series of a tangentially relevant variable.

In keeping with the above remarks, in this simulation the population of the country is broken down into four sociological groups which are broadly described as "commercial white," \* "agricultural white," mestizo, and Indian. It should be emphasized that these are sociological and not racial categories. A full blooded Indian could easily have the social status of a "commercial white."

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\* This category will be assumed to include resident foreigners.

Another way of looking at this breakdown sociologically would be:

	Low Status	High Status
modern	mestizo	"commercial white"
traditional	Indian	"agricultural white"

## 2. Gross National Product or Expenditure

In the following sections we return to the more or less traditional path of listing the various identities and relations of a national accounting scheme. These are integrated with the breakdown into social classes and a scheme for social mobility. The economic system may be regarded as providing a framework upon which the more difficult to identify and quantify social and political factors can be added.

We begin by presenting the major expenditure components of gross national product. These show the basic components of GNP (consumption, investment, government, etc.), and the relationship of GNP to other important concepts such as personal income and disposable income. Functional relationships relating the major expenditure components to relevant variables are also set up. These may be interpreted as behavior and policy relations.



Gross national product is computed separately both from the expenditure and income sides of the balance sheet. In general these calculations do not match and the discrepancy is explained in terms of changes in the price level. Details of this are discussed in section 4.3.

- Let  $Y_t$  = Gross national product in period  $t$  in current' currency
- $I_t$  = Gross private and public capital formation in current currency
- $C_t$  = Total consumption in current currency
- \*  $G_{2,t}$  = Government expenditures on health, education, and welfare in current currency
- $G_{3,t}$  = Other government current expenditures
- $X_t^1$  = Exports of goods and services in current currency (f.o.b.)
- $M_t^1$  = Imports of goods and services in current currency (c.i.f.)
- $M_t^{2,1}$  = Factor income received.
- $X_t^{2,1}$  = Factor income paid + .

We have the accounting identity defining GNP from the viewpoint of expenditures:

$$(1) \quad Y_t = I_t + C_t + G_{2,t} + G_{3,t} + (X_t^1 - M_t^1) + (M_t^{2,1} - X_t^{2,1})$$

---

\* See 2.2.1 for further discussion and breakdown

+ From here on all references are to current domestic currency unless stated otherwise.

Equation (2) specifies that total personal income equals Gross National Product minus depreciation, retained earnings, and indirect business taxes and plus transfer payments.\* This may also be regarded as the sum of factor payments augmented by transfers.  $\Pi_{6,t}$  is profits of government-owned enterprise.

$Y_t^1$  = Total personal income

$D_t$  = Depreciation (Capital Consumption Allowances)  
in constant currency

$T_t$  = Total taxes

$T_t^1$  = Indirect taxes

$Y_t^0$  = Gross domestic product in constant currency

$P_t$  = Domestic price level

$T_t^2$  = Direct taxes on business

$R_t$  = Net transfer payments to individuals

$CS_t$  = Retained earnings

$\Pi_t$  = Total profit

$\Pi_{i,t}$  = Profit of the  $i^{\text{th}}$  group ( $i = 1, \dots, 6$ )

$$(2) \quad Y_t^1 = (Y_t^0 - D_t)P_t + (M_t^{2,1} - X_t^{2,1}) - T_t^1 + R_t - T_t^2 - CS_t - \Pi_{6,t}$$

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\* We assume that international transfers to individuals are negligible for these countries.

where the first two terms express gross national product calculated from the production side of the accounts.

We consider the population to be broken down into four socio-economic categories in the countries of our interest, (for example: Peru, Bolivia, Ecuador, Mexico, and Guatemala). They are: "Commercial" Whites ( $i = 1$ ), "Agricultural" Whites ( $i = 2$ ), Mestizos ( $i = 3$ ), and Indians ( $i = 4$ ). The color divisions are not actually racial in South and Central America. An approximate criterion is that a detribalized Indian becomes a mestizo. A mestizo who attains an appropriate degree of wealth and education may become a white. These divisions will be discussed in further detail in subsequent sections. The subscript 5 is used to denote foreign enterprise in the country;\* while 6 refers to government.

$$Y_{i,t}^1 = \text{Personal income of the } i^{\text{th}} \text{ socio-economic category}$$

We have the identity:

$$(3) \quad Y_t^1 = Y_{1,t}^1 + Y_{2,t}^1 + Y_{3,t}^1 + Y_{4,t}^1 .$$

In equation (4) disposable personal income ( $Y_t^2$ ) is defined as personal income minus taxes levied on personal income.

$$Y_t^2 = \text{disposable income}$$

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\* The income of foreign residents is assumed to be included in the first class.

$T_t^3$  = Direct taxes on households

$Y_{i,t}^2$  = Disposable income of  $i^{\text{th}}$  socio-economic category

$$(4) \quad Y_t^2 = Y_t^1 - T_t^3 = Y_{1,t}^2 + Y_{2,t}^2 + Y_{3,t}^2 + Y_{4,t}^2 .$$

### 2.1 Consumption

All of the above equations have been accounting identities.

At this point we must specify the first behavioral relationships.

$C_t$  = total consumption

$C_{i,t}$  = consumption of the  $i^{\text{th}}$  group

$N_t$  = total population

$N_{i,t}$  = population of the  $i^{\text{th}}$  socio-economic group.

$\eta_j$  = random variables

Equation (5) is a mere accounting identity summing consumption in the different classes.

$$(5) \quad C_t = \sum_{i=1}^4 C_{i,t} .$$

$$(6) \quad C_{i,t} = \alpha_{i,1} N_{i,t} + \alpha_{i,2} Y_{i,t}^2 + \eta_j$$

In equation (6) we have assumed as a first approximation that per capita consumption can be described as a linear function of per capita disposable income. If we believed that (6) held for

all incomes and over considerable lengths of time we would run into difficulties with assigning values of  $\alpha_{i,1} > 0$  as this would leave open the possibility that individuals with no income could continue to consume indefinitely.

The  $\alpha_{i,1}$  are not dimension free parameters but must be measured in terms of {money/people} . As an economy develops one might wish to introduce credit effects on consumption.

$$(6a) \quad S_{i,t} = Y_{i,t}^2 - C_{i,t} ; \text{ where}$$
$$S_{i,t} = \text{Savings of the } i^{\text{th}} \text{ group.}$$

## 2.2 Government

Government activities may be grouped, for national accounts purposes, into two broad categories: expenditures, which represent an allocation of part of gross national product, and taxes and transfers, which are, of course, means of redistributing income. The next section takes up government expenditures and, although taxes and transfers do not necessarily belong here, they are taken up in Section 2.2.2.

### 2.2.1 Government Expenditures

Government expenditures in our model are divided into four categories , which we believe are particularly relevant for a development study: gross investment; health, education, and welfare; transfers; and other expenditures. These are denoted:

$$G_{1,t} = \text{gross investment by government}$$

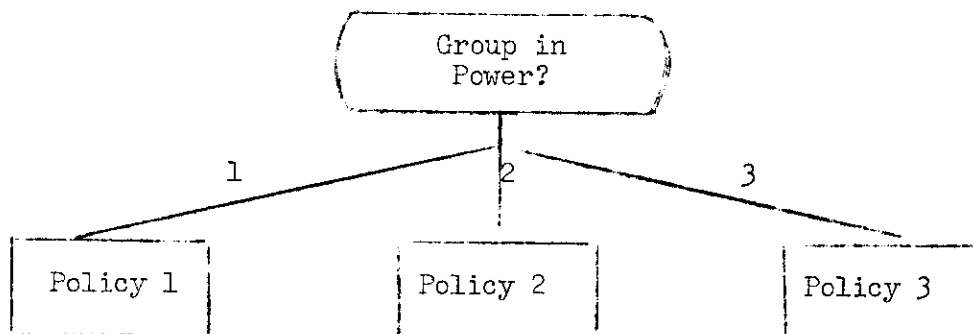
$G_{2,t}$  = government expenditures on health, education  
and welfare

$G_{3,t}$  = other government current expenditures

$R_t$  = transfers and subsidies

The breakdown of government expenditures will be determined by the group in power and represents one of the politico-economic aspects of this model which needs to be investigated in detail. For example, it appears to be worthwhile to distinguish between two types of "revolutionary" change in a Latin American state; the first a change in the "palace guard" which might result in a shift in the biases between landed and commercial or industrial interests; the second, a deep social revolution such as that of Mexico in 1911 or the recent Cuban revolution.

In the content of this model we eventually will wish to introduce political control alternatives explicitly. The method for doing this is indicated in the flow diagram below. However, we limit ourselves by the assumption that the governmental group in power will remain fixed over the period under consideration.



Until a satisfactory first approximation for adding political features has been made, two approaches are adopted for the next three behavioral equations.

The first is to leave them open for exogenous inputs. This procedure can be defended in several ways.

In many countries government policy may be a critical item in the determination of growth and welfare. Given the current state of social, political and economic information and the difficulties in constructing models of politico-economic process, in checking previous histories, and in testing the effects of policies the use of historical data or of plans both simplifies problems of validation and permits experimentation with planning procedures.

Once more it must be stressed that such an approach is not offered as an alternative to econometric methods. In the construction of models for development, planning, and the control of economies it appears to be desirable to distinguish between behavioral relations which are or are not amenable to economic

analysis, depend upon large numbers of independent actions and/or exhibit some special features leading to stability in structure over a reasonable period of time. Those which exhibit these properties are amenable to an econometric approach. Those which do not, yet may be important in describing growth processes, must be approached differently.

Reasons for opposing this approach are that it introduces too many additional degrees of freedom into the model and increases the complexity and unwieldiness of the input.

The second approach is to ignore all the caveats just made and to try to specify behavioral equations.

As a simple first approximation we assume that government gross domestic capital formation is given by:

$$(7) \quad G_{1,t} = \beta_1 + \beta_2 Y_{t-1} + \beta_3 [M_{t,6}^{2,2} + M_{t,6}^{2,3}]$$

Here government investment is a simple linear function of lagged gross national product and transfers and long term capital flows from abroad.

$$M_{t,6}^{2,2} = \text{Net capital inflows to government}$$

$$M_{t,6}^{2,3} = \text{Net transfer payments from abroad to government}$$



Government expenditures on health, education, and welfare are assumed to depend on gross national product and total population during the previous period. This allows for an increase in health, education and welfare expenditures as either population or GNP rises. This is shown below in equation (8) .

$$(8) \quad G_{2,t} = \beta_3 + \beta_4 Y_{t-1} + \beta_5 N_{t-1}$$

Other government expenditures are assumed to grow with population. (It is of interest to note that the Ecuador Central Bank bulletin recommends that the government adjust its expenditures to accord with revenue <sup>4</sup> ).

$$(9) \quad G_{3,t} = \beta_6 N_t P_{t-1}$$

In Ecuador there is considerable earmarking of taxes with the result that automatic transfer of revenues to various of the 18 provinces, 97 municipalities, and around 700 other autonomous entities dilutes the Central Government's abilities to use the national public resources.

#### 2.2.2. Government Revenue and Transfers

Revenues can be considered in three broad headings:

Direct taxes,

Indirect taxes, and

Non-tax revenue.

Under the first there are income tax, inheritance and gift taxes and export taxes. Indirect taxes include excise taxes, consumption and sales taxes and import taxes. The non-tax revenues include income from property and charges for services.

In this relatively simplified model only four taxes, two direct and two indirect, are considered. They are personal income tax, profits, indirect business taxes other than tariffs, and tariffs.

The basic tax identities are given below:

$$(10) \quad T_t = T_t^1 + T_t^2 + T_t^3$$

$$(11) \quad T_t^1 = T_t^{1,1} + T_t^{1,2}$$

$T_t$  = total tax revenue

$T_t^1$  = indirect business taxes

$T_t^2$  = taxes on profits

$T_t^3$  = personal income taxes

$T_t^{1,1}$  = indirect business taxes, except tariffs

$T_t^{1,2}$  = tariffs

Equation (10) gives total tax revenue as the sum of indirect business taxes ( $T_t^1 = T_t^{1,1} + T_t^{1,2}$ ) and income taxes ( $T_t^2 + T_t^3$ ). In equations (12) and (13)  $T_t^{1,1}$  is assumed proportional to gross national product, and  $T_t^{1,2}$  to imports of goods and services:

$$(12)' \quad T_t^{1,1} = \beta_7 Y_t$$

$$(13) \quad T_t^{1,2} = \beta_8 M_t^1$$

The tax on profits is assumed proportional to profits earned in the private sector, except that it does not become negative:

$$(14) \quad T_{i,t}^2 = \max \left[ \beta_9 \pi_{i,t}, 0 \right] \quad i = 1, 2, 3, 5.$$

$\pi_{6,t}$  are profits of government owned enterprise and hence are not taxed.

Personal income taxes are assumed proportional to personal income; however, the constant of proportionality will not be the same for all four socio-economic groups; there are progressive tax rates, not fully reflected in this approximation.

$$(15) \quad T_{i,t}^3 = \alpha_{i,3} Y_{i,t}^1$$

$$(16) \quad T_t^3 = \sum_{i=1}^4 T_{i,t}^3$$

It is clear from equation ( 2) that personal income includes the after tax profits of the four socio-economic groups. Thus, profits are double-taxed in this model. Tax laws vary considerably from country to country on this point.

Transfers to the government from abroad are assumed exogenous to the model, being determined by the foreign policies

of the country and of other nations. Net transfers scheduled by the government to the four socio-economic groups are given below:\*

$$(17) \quad R_t = \sum R_{i,t}$$

$$(18) \quad R_{i,t} = \alpha_{i,4} Y_{t-1} - \alpha_{i,t} \min [(Y_{t-1} - Y_{t-2}), 0]$$

Equation (18) by definition cannot have a negative value. This equation should be considered as a key policy control and should either be exogenous or merits careful empirical investigation. Here a crude approximation has been used linking transfers to GNP with a simple countercyclical term. For most countries this fits the facts poorly as both specific changes in policy and reporting methods can be observed in the series.

Total government expenditure is:

$$(19) \quad E_t = G_{1,t} + G_{2,t} + G_{3,t} + R_t + J_t ; \text{ where}$$

$J_t$  = Interest on public debt,

and total revenues are:

$$(20) \quad REV_t = T_t + M_{t,6}^{2,3} + \Pi_{6,t} ; \text{ where}$$

$E_t$  = government total expenditures

$REV_t$  = government total revenues

$M_{t,6}^{2,3}$  = Net transfers and payments to the government from abroad

$R_{i,t}$  = transfers by the government to the  $i^{\text{th}}$  socio-economic group

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\* Subsidies should be accounted for by a separate equation if they are large.

and:

$$(21) \quad J_t = \beta_{10} \text{DET}_t \quad ; \text{ where}$$

$\text{DET}_t$  = government debt

$$(21a) \quad \text{SC}_t = \text{REV}_t - \text{E}_t + \text{G}_{1,t} \quad ; \text{ where}$$

$\text{SC}_t$  = Surplus on current account

$$(21b) \quad \text{DEF}_t = \text{SC}_t - \text{G}_{1,t} \quad ; \text{ where}$$

$\text{DEF}_t$  = government surplus

$$(21c) \quad \text{BOR}_t = \text{BOR}_{1,t} + \text{BOR}_{2,t} + \text{M}_{t,6}^{22} \quad ; \text{ where}$$

$\text{BOR}_t$  = Total borrowing

$\text{M}_{t,6}^{22}$  = Foreign "

$\text{BOR}_{1,t}$  = Borrowing from banking system\*

$\text{BOR}_{2,t}$  = Borrowing from out of the banking system

Without detailed information it is easiest to treat foreign borrowing as exogenous. Furthermore in many planning uses foreign borrowing is a critical decision variable.

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\* Net increase in the claims of the banking system.

$$(21d) \quad \text{DET}_t^1 = \text{DET}_{t-1}^1 \left( \frac{\text{FEXCH}_t}{\text{FEXCH}_{t-1}} \right) + M_{t,6}^{22} (\text{FEXCH}_t) ; \text{ where}$$

$\text{DET}_t^1$  = Foreign debt (in dollars, U.S.)

$\text{FEXCH}_t$  = Foreign exchange rate

$$(21e) \quad \text{DET}_t^{21} = \text{DET}_{t-1}^{21} + \text{BOR}_{1,t} ; \text{ where}$$

$\text{DET}_t^{21}$  = Bank held domestic debt. \*

$$(21f) \quad \text{DET}_t^{22} = \text{DET}_{t-1}^{22} + \text{BOR}_{2,t} ; \text{ where}$$

$\text{DET}_t^{22}$  = Non-bank held domestic debt.

$$(21g) \quad \text{DET}_t^2 = \text{DET}_t^{21} + \text{DET}_t^{22} ; \text{ where}$$

$\text{DET}_t^2$  = Total domestic debt.

A behavior or policy equation is needed to explain the relative sizes of bank and non-bank government borrowing.

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\* Net outstanding claims. Liabilities of the government to the banking system net of its deposits.

### 2.3 Investment in Physical Capital

Eight special symbols which are used below are now noted.

They are:

$$\hat{G}_{1,t}, \hat{G}_{2,t}, \hat{G}_{3,t}, \hat{R}_t, \hat{I}_{1,t}, \hat{I}_{2,t}, \hat{I}_{3,t}, \text{ and } \hat{I}_{5,t} .$$

Their detailed definitions are given in 4.2.3; briefly they account for possible leakages of funds.  $\hat{G}_{1,t}$  stands for "effective" government investment as contrasted with  $G_{1,t}$  which stands for government investment; and similarly for the others.

$$(22) \quad \hat{G}_{1,t} = (1 - \alpha_{1,22} - \alpha_{2,22}) G_{1,t} .$$

There are similar equations for all of the others.

In this section we consider gross investment by the four socio-economic groups and by foreign enterprises. Government investment was discussed above in Section 2.2.1. One feature related to government investment needs to be specified. That is capital stock.

$$(23) \quad K_{6,t} = (1 - \alpha_{6,8}) K_{6,t-1} + \hat{G}_{1,t} / P_t .$$

$$K_{i,t} = \text{the capital stock owned by members of the } i^{\text{th}} \text{ socio-economic group}$$

The poorness of capital stock estimates is well known even in highly developed countries. Nevertheless, even a crude approximation is of value.

2.3.1. Gross Investment by Commercial Whites

Without specifying the country and combining some casual empiricism with sociological and economic considerations one form of an investment function for the commercial group which might be worth investigating is given in (24a):

$$(24a) \quad I_{1,t} = \text{Max} \left[ 0, \left\{ \alpha_{1,6} \left( \frac{Y_{t-1} - Y_{3,t-1}^2}{P_{t-1}} \right) - \left( \frac{Y_{t-2} + Y_{3,t-2}^2}{P_{t-2}} \right) + \alpha_{1,7} \left( \frac{Y_{3,t-1}^2}{P_{t-1}} - \frac{Y_{3,t-2}^2}{P_{t-2}} \right) + \alpha_{1,8} (K_{1,t-1})(P_{t-1}) \right\} \right]$$

It is the sum of two accelerator terms and a depreciation term. The first accelerator term involves changes in GNP leaving out changes in mestizo disposable income, while the second accelerator term involves changes in mestizo disposable income. It is assumed that change in mestizo income is of particular import to investment in an economy as its growth represents the possible expansion of a lower middle class market. A serious drop in this income may also be an important factor in the growth of political instability.

The determination of reasonable investment functions is obviously one of the most critical empirical problems in the construction of this type of model. In the case of Ecuador, bank policy, the availability of foreign long term finance and the size of the export market appear to have been dominant factors. "Betting on bananas" appears to have been a policy followed recently.



In view of the considerations noted above an alternative gross investment function which we use is suggested in (24b):

$$(24b) \quad I_{1,t}^1 = \text{Max} \left[ \left( \alpha_{1,6} X_{t-1}^1 + \alpha_{1,8} K_{1,t-1} \right) P_{t-1} - \alpha_{1,32} \beta_8 \right], GC_t^1, BC_t^1$$

where  $GC_t^1$  = Government direct control limit on private capital imports

$BC_t$  = Credit limit

$I_{1,t}^1$  = Private sector imported capital goods

This states that investment is primarily dependent upon previous exports modified to some extent by import tariffs and constrained by credit limitations and direct governmental controls. In some countries with imperfectly developed capital markets limits may play a more important role than the rate of interest.

Equation (24b) suggests a priority in investment considerations; the amounts to be imported are considered first here with a fixed proportion assumed between local and foreign procurement. If substitutability is low this is a reasonable assumption. Furthermore an investor will try not to be stuck with an almost completed project to find that he cannot obtain some vital overseas parts due to restrictions.

$$(24c) \quad I_{1,t}^2 = \alpha_{1,33} I_{1,t}^1 \quad ; \quad \text{where}$$

$I_{1,t}^2$  = Capital goods of domestic origin purchased by local enterprise.

$$(24d) \quad I_{1,t} = I_{1,t}^1 + I_{1,t}^2 .$$

Depreciation in fixed currency is given by:

$$(25) \quad D_{i,t} = \alpha_{i,8} K_{i,t-1}$$

Capital stock for the commercial class is given by:

$$(26) \quad K_{1,t} = (1 - \alpha_{i,8}) K_{1,t-1} + \hat{I}_{1,t} / P_{1,t} .$$

The description of investment leaves out terms of trade, import and export elasticities, all of which are obviously important in a reasonably competitive market, but are far less so in international oligopolistic or oligopsonistic markets. The implications of the model here are that for at least some commodities one sells what one can at whatever prices one can obtain above variable costs. This description could be little more than part of a mythology concerning trade in underdeveloped countries; at least we may explore its implications in a formal model here with relative ease.

### 2.3.2 Gross Investment by Agricultural Whites

Investment in this group is taken to depend on disposable income within the group. For these people investment may be more a matter of status than of rational calculation. It has been suggested, for instance, that the social role of "patron" is in some areas still of

importance. Thus, the additions to the hacienda, finca, or estancia may be a manifestation of the role rather than a pure economic act. Investment would be predominantly internal.

Once more it is stressed that even though the following equation might be regarded as a caricature of the "patron" a valuable role of a simulation is to serve as a method for formalizing, categorizing, and testing the folklore.

$$(27) \quad I_{2,t} = \alpha_{2,6} + \alpha_{2,7} Y_{2,t-1}^2 + \alpha_{2,8} (K_{2,t-1}) (P_{t-1})$$
$$\quad \text{if } \frac{Y_{2,t-1}^2}{N_{2,t-1}} \geq \beta_{31} P_{t-1} ,$$
$$\quad = \alpha_{2,8} (K_{2,t-1}) (P_{t-1})$$
$$\quad \text{if } \frac{Y_{2,t-1}^2}{N_{2,t-1}} < \beta_{31} P_{t-1}$$

Capital stock for the second socio-economic group is given by:

$$(28) \quad K_{2,t} = (1 - \alpha_{2,8}) K_{2,t-1} + \hat{I}_{2,t} / P_t$$

### 2.3.3 Gross Investment by Mestizos and Indians

In general, mestizos have little surplus for investment; nevertheless we assume that:

$$(29) \quad I_{3,t} = Y_{3,t-1}^2 - C_{3,t-1}$$

This indicates that they invest their savings, but with a time lag. At first glance and historically, this factor may be small yet as far as policy, social mobility, and political stability it may be of considerable importance.

There is the standard accounting equation for capital stock:

$$(30) \quad K_{3,t} = (1 - \alpha_{3,8}) K_{3,t-1} + \hat{I}_{3,t} / P_t$$

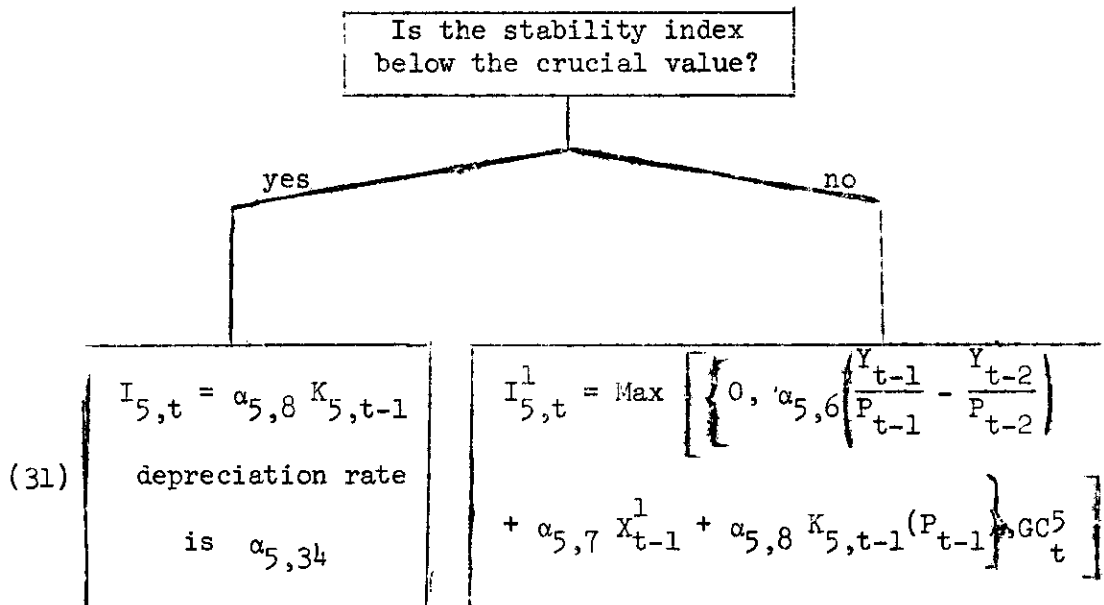
Our initial assumption is that investment by the Indian sector is negligible. In this model we omit further consideration of it.

#### 2.3.4 Gross Investment by Foreign Firms

The final category of investment is investment by foreign enterprises. Foreign investors are probably more ready, willing, and able to pull out of a country when the country appears to be heading toward social or political instability than are the local citizens. One simple way to represent this is with a "domestic stability index." The index could be a function of the length of time the present government has been in power, the recent growth rate of gross national product, and the growth rates of real wages for the various socio-economic groups. Usually the index will be the higher, and the country the more stable, the higher are these variables. If the index is below a crucial value, the foreign investors seek to pull out at the fastest possible rate, which is the maximum depreciation rate.\* The diagram below represents the investment decision for foreign investors:

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\* It is possible that some assets can be transported out of the country, but in general this is unlikely.



$I_{5,t}$  = the rate of gross investment of foreign nationals

$GC_t^5$  = government limit on import of capital goods

Investment is broken into imports and local procurement

$$(31a) \quad I_{5,t}^2 = \alpha_{5,33} I_{1,t}^1$$

$$(31b) \quad I_{5,t} = I_{5,t}^1 + I_{5,t}^2$$

$$(32) \quad K_{5,t} = (1 - \alpha_{5,7}) K_{5,t-1} + \hat{I}_{5,t} / P_t$$

Thus if the index is below the crucial value, gross investment may equal net investment and is  $-\alpha_{5,34} K_{5,t-1}$  in constant currency.

On the other hand, if the index is sufficiently high, investment is determined by depreciation and the change in gross national product

between the previous period and two periods prior; it is the greater of zero or the amount calculated from these considerations. We note that there are two depreciation rates above,  $\alpha_{5,8}$  and  $\alpha_{5,34}$ .

Through hard driving and poor maintenance, capital can be made to depreciate faster than it would with careful use and maintenance. The actual rate of depreciation for a capital good will depend on the cost of replacement, expected returns on it and the cost of various levels of maintenance. A foreign investor wishing to pull out of a country due to the instability of the political situation may expect expropriation or wholesale destruction of his investment. If so, he may want to work his capital as hard as possible without spending anything on it until it has no value left.  $\alpha_{5,34}$  is the maximal rate of depreciation relevant to when the capital is used up as quickly as possible.  $\alpha_{5,8}$  is the "normal rate", applicable when the investor expects to stay in the country indefinitely. It will be seen later that the stability index has a bearing on international capital movements.

In many debates on policy the role of foreign investment is usually discussed. The actual nature of the foreign investment function may depend heavily upon explicit laws, special economic opportunities and the faith placed in the political and economic ambiente.

An interesting use of a model of this type is to carry out gaming exercises for the exploration of policy. A way in which the simulation can be used for gaming is to have a set of players select foreign private investment and other capital flows exogenously, while the other team controls tariffs and other aspects of government policy.

#### 2.4 Exports and Imports of Goods and Services

The demand for exports is assumed to be autonomous, being determined by conditions abroad which are essentially unaffected by developments in the country. In some countries such as Ecuador<sup>+</sup> or Guatemala four or five items account for 75-90% of exports. A reasonably satisfactory model for exports could be constructed by including some features of prices and demands for these specific commodities.<sup>3/</sup> Even this disaggregation is not completely satisfactory as it is in general the case that the commodities traded are highly subjected to oligopolistic influence. Hence, a good model of foreign trade needs to take this into account.

We treat the demand for exports as exogenous. Exports could, of course, be considerably less than the demand for exports; however, for crops such as bananas or rice, it is possible to increase production in a year.

From the viewpoints of operational gaming, planning, and growth studies it is desirable to experiment with alternative conditions on exports and to carry out separate studies to locate the factors to which exports are sensitive.

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<sup>+</sup> In Ecuador, for example, bananas, coffee, cacao and rice form more than 90% of exports.

Imports depend upon consumption, investment, foreign transfers, and credit conditions as well as export and import price levels, tariffs, and direct controls and bank policy. These are reflected in equation (34). The term  $G_{1,t}^1$  may be very small unless the government has important nationalized sectors of the economy.

$$(33) \quad X_t^1 = \text{an exogenous series for exports}$$

$$(34) \quad M_t^1 = I_{1,t}^1 + I_{5,t}^1 + C_t^1 + G_{1,t}^1 ; \text{ where}$$

$$C_t^1 = \text{imports generated by consumption}$$

$$G_{1,t}^1 = \text{government investment imports.}$$

As with investment, we assume that part of consumption calls for imports. This is often directly controlled and results in large contraband operations which are not fully reflected here.

$$(34a) \quad C_t^1 = \beta_{11} C_t$$

$$(34b) \quad C_t = C_t^1 + C_t^2 .$$

### 3. Human Capital

A very important ingredient in successful growth for an underdeveloped country, indeed for any country, is investment in a healthy, educated population. The importance of attempting to measure the effect of health, education and welfare (HEW) expenditures on productivity is obvious, as are the difficulties in doing so.



Below we sketch two models for the measurement of human capital. The first is an input-output model which we do not utilize at present. It is included because it is our hope that this basic approach may eventually prove feasible and fruitful. The second approach, which is utilized for the first approximation, includes a method \* for converting the labor force of each socio-economic group into standard labor units which are comparable between classes. The number of standard labor units into which a member of the labor force converts depends on the HEW expenditures made on members of his group. While this method leaves something to be desired, it does represent a beginning at the the difficult task of measuring labor productivity as a function of HEW.

### 3.1 Input-Output Model

In Table I below appears a simple input-output scheme. Each column is an activity. The negative coefficients are outputs and the positive, inputs. Thus, looking at the first activity, it shows that if we put in one infant,  $\gamma_{6,1}$  of social and medical resources and  $\gamma_{7,1}$  of education resources, we will get as outputs one child and  $\gamma_{8,1}$  of labor.

	Infants :	+1			
	Children :	-1	+1		
	Young Adults :		-1	+1	
	Middle Aged :			-1	+1
	Old :				-1 +1
Costs: Social,Medical:	$\gamma_{6,1}$	$\gamma_{6,2}$	$\gamma_{6,3}$	$\gamma_{6,4}$	$\gamma_{6,5}$
Costs: Education :	$\gamma_{7,1}$	$\gamma_{7,2}$	$\gamma_{7,3}$	$\gamma_{7,4}$	$\gamma_{7,5}$
Labor Output:	$\gamma_{8,1}$	$\gamma_{8,2}$	$\gamma_{8,3}$	$\gamma_{8,4}$	$\gamma_{8,5}$

TABLE I

\* This method was primarily suggested and devised by James Friedman.

We have divided the life span into five stages here and have implicitly assumed that at each stage there will be a linear relationship linking the labor output (measured in some standard unit) with the HEW inputs.

In order to study the effectiveness of the HEW programs we might wish to expand the number of activities shown in Table I to account for different inputs in education, for example. As a good second approximation four social groups, each portrayed by five activities, should provide sufficient information for the estimation of human capital.

If we are able to measure labor output in terms of a standardized dollar and if we are willing to specify a discount factor in the economy we can calculate a value for an individual in the t-th age group.

$$V_t = \sum_{i=t}^5 \rho^{i-t+1} (\gamma_{8,i} - \gamma_{6,i} - \gamma_{7,i})$$

where

$V_t$  = the human capital value of an individual in age group t

$\rho$  = discount rate

$\gamma_{k,i}$  = input output coefficient

### 3.2 The Current Model

The model which we look at in this section averages over all members of a socio-economic group identically, regardless of age and sex. It would not be difficult to expand it to keep track of the population within each socio-economic group by age groups. The steps in the model are two: 1) The size of the population is computed, taking into account births, deaths, and transfers from one class to another. 2) The labor force is converted into standard labor units, taking into account the per capita HEW expenditures made in recent years.

Let:

$N_t$  = Total population in time  $t$

$N_{i,t}$  = population of  $i$ -th group in time  $t$

$A_t$  = deaths in total population during period  $t$

$A_{i,t}$  = deaths in  $i$ -th group

$B_t$  = births in total population

$B_{i,t}$  = births in  $i$ -th group

$F_{1,t}$  = number of people who transfer from 3rd  
(mestizo) group to group 1 (Commercial White)

$F_{3,t}$  = number who transfer from 4th (Indian)  
group to 3rd

Some basic bookkeeping relationships which involve the variables defined above are:

$$(35) \quad N_t = N_{t-1} + B_{t-1} - A_{t-1}$$

$$(36) \quad N_{1,t} = N_{1,t-1} + B_{1,t-1} - A_{1,t-1} + F_{1,t-1}$$

$$(37) \quad N_{2,t} = N_{2,t-1} + B_{2,t-1} - A_{2,t-1}$$

$$(38) \quad N_{3,t} = N_{3,t-1} + B_{3,t-1} - A_{3,t-1} - F_{1,t-1} + F_{3,t-1}$$

$$(39) \quad N_{4,t} = N_{4,t-1} + B_{4,t-1} - A_{4,t-1} - F_{3,t-1}$$

Equation (35) gives total population in period  $t$  as population in period  $t-1$  plus births and minus deaths during period  $t-1$ . Equations (36) through (39) are the analogous equations for each group. Two values of inter-group transfer are allowed for in equations (42) and (43). The relationships governing births and deaths are given below:

$$(40) \quad A_{i,t-1} = \alpha_{i,30} N_{i,t-1}$$

$$(41) \quad B_{i,t-1} = \alpha_{i,31} N_{i,t-1}$$

Transfers from one class to another are considered to occur when a mestizo acquires sufficient wealth and position to be, in fact, a member of the commercial white community; or when an Indian turns outward from his tradition-bound environment and tries to get along in the more market-oriented part of society. The two transfer equations

are:

$$(42) \quad F_{1,t-1} = \alpha_{1,9} N_{3,t-1} \left( \frac{N_{1,t-1} \cdot N_{3,t-1}}{(N_{1,t-1} + N_{3,t-1})^2} \right) \left( \alpha_{1,10} \frac{Y_{3,t-1}^2}{N_{3,t-1}} + \alpha_{1,11} \frac{H_{3,t-1}}{N_{3,t-1}} \right) \frac{1}{P_{t-1}}$$

$$*(43) \quad F_{3,t-1} = \alpha_{3,9} N_{4,t-1} \left( \frac{N_{3,t-1} \cdot N_{4,t-1}}{(N_{3,t-1} + N_{4,t-1})^2} \right) \left( \alpha_{3,10} \frac{Y_{4,t-1}^2}{N_{4,t-1}} + \alpha_{3,11} \frac{H_{4,t-1}}{N_{4,t-1}} \right) \frac{1}{P_{t-1}}$$

In both these equations, the first expression in brackets is the proportion of the combined population of the two groups which is in one of the classes multiplied by the proportion in the other. This proportion is modified by a parameter,  $\alpha_{i,9}$ , and by an expression involving per capita disposable income for the group and HEW expenditures. The result is modified by the size of the group from which people will transfer. The equations are modified for the effect of price level. It is assumed that as HEW and per capita income rise, the numbers which will transfer rise.

---

\* A consistency check requires that  $\alpha_{3,9}[1/2] [\dots] \leq 1$ . Otherwise the size of population transfer would be greater than the population.

The next step is to determine the labor force for each group and then convert into standard labor units.

$$(44) N_{i,t}^1 = \alpha_{i,12} N_{i,t}$$

$$(45) N_{i,t}^2 = e^{\left( \alpha_{i,13} - \frac{\alpha_{i,14}}{RH_{i,t}^1 + \alpha_{i,15}} \right)} N_{i,t}^1$$

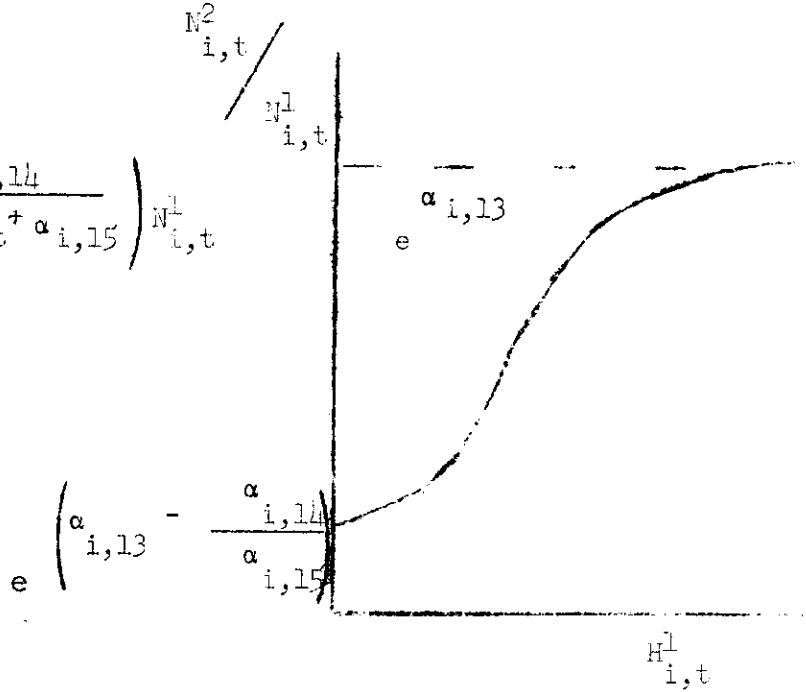


Figure 1

where

$H_{i,t}$  = total effective HEW spent on group i

$N_{i,t}^1$  = labor force of group i

$N_{i,t}^2$  = number of standard labor units represented by the labor force of group i

$RH_{i,t}^1$  = real per capita HEW on group i, summed for  $\beta + 1$  years

$\alpha_{i,12}$  is the proportion of the i-th group which is in the labor force. \*

The general shape and characteristics of equation (45) are illustrated in Figure 1. The minimum value of  $N_{i,t}^2$ , when  $RH_{i,t}^1$  is zero, is

\* Further study is needed to consider the meaning of implicit unemployment in an underdeveloped economy.

$N_{i,t}^1 e^{\alpha_{i,13} - \alpha_{i,14}/\alpha_{i,15}}$ . This is constrained to be non-negative.

HEW shows increasing, then decreasing returns. The marginal product of HEW in terms of labor units is always positive, although it is asymptotic to zero and total labor units are asymptotic to

$$N_{i,t}^1 e^{\alpha_{i,13}}$$

The derivation of  $RH_{i,t}^1$  remains:

$$(46) \quad H_{i,t} = \alpha_{i,16} + \alpha_{i,17} C_{i,t} + \alpha_{i,19} \hat{G}_t^2$$

$$(47) \quad \sum_{i=1}^4 \alpha_{i,19} = 1$$

$$(48) \quad RH_{i,t} = H_{i,t} / P_{t-1}$$

$$(49) \quad RH_{i,t}^1 = \sum_{\theta=t-\beta_{16}}^t \frac{RH_{i,\theta}}{N_{i,\theta}} = RH_{i,t-1}^1 - \frac{RH_{i,t-\beta_{16}-1}}{N_{i,t-\beta_{16}-1}} + \frac{RH_{i,t}}{N_{i,t}}$$

According to equation (46), the HEW expenditures on the  $i^{\text{th}}$  group are a linear function of the consumption expenditures of the group and of government HEW expenditures. This implies in particular that we should count private expenditures on education and health as affecting productivity of the group spending. Equation (47) is merely a constraint to assure that the whole of  $\hat{G}_t^2$  is parcelled out among the four groups.

Equation (48) changes the current HEW expenditure into fixed currency terms. Equation (49) is the sum of per capita HEW in group  $i$  for the current year and the  $\beta_{16}$  preceding years.

It must be noted that the definition of HEW expenditures presents several difficult problems both in theory and in national accounts classification.

#### 4. Production and Distribution

In this section the treatment of production and distribution is outlined. The production side of the model utilizes an aggregate Cobb-Douglas production function. The assumptions about distribution are a variant of traditional marginal productivity theory.

The program includes the more general C.E.S. production function for purposes of experimentation; however, the worth of this extension is doubtful and discussion here is limited to the Cobb-Douglas.<sup>4/</sup> In any expansion of a model of this type a natural step would be to replace this total aggregate by a small input-output table.

##### 4.1 Production

We assume the existence of three factors: capital, labor, and the "institutional" factor. The third of these is not represented directly in the production function, and is discussed below.



Let:

$Y_t^0$  = gross domestic product in constant currency

$$(50) \quad P_t Y_t^0 = Y_t - M^{2,1} + X^{2,1}$$

Equation (50) contains an accounting identity which defines gross domestic product as the sum of all goods and services produced within the country. Equation (51) expresses gross domestic product, as an output, related to the labor force and capital stock as inputs. This "aggregate production function" we have assumed to be a Cobb-Douglas function. +

$$(51) \quad Y_t^0 = \beta_{17} K_{t-1}^{\beta_{18}} (N_t^{2,1})^{1-\beta_{18}} e^{\beta_{32} t} \quad (0 < \beta_{18} < 1)$$

$$(52) \quad K_t = \sum_{i=1}^6 K_{i,t}$$

Equation (52) is an accounting equation for the total worth of capital stock. It is evident that for many policy purposes it would be desirable to disaggregate the production function and consider at least an agricultural and other sector separately.

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+ modified by the term  $e^{\beta_{32} t}$  to account for technological progress

#### 4.2 Distribution of Product

We assume that generally labor and capital are paid in proportion to their marginal productivity with the exception of the Indian sector of the population where institutional rigidities have an influence. In many Latin American countries there are extreme rigidities in payments to government employees and in places such as Chile, Argentina, and Uruguay union and government negotiations play an important role.

An allocation of income is also made to an "Institutional Administrative" factor. The appropriate licenses, administrative blessings, cooperation of local authorities, adequate protection against institutional delays and so forth all are part of a production process and may be regarded as very valuable factors. They are, in general, resources with increasing returns to scale. Essentially it costs the mayor little extra to sign additional permissions to build, but each permission is nevertheless needed as a factor. There is possibly an upper bound or a capacity beyond which an individual cannot go without dipping too heavily into his "political capital" (a quantity we do not attempt to measure at this point); hence, a given administrative system may be regarded as having a capacity in the same as does a bridge or railway system.\*

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\* Up to a point he may actually be increasing his "political capital" when he grants permissions.

#### 4.2.1 Wages

The returns to labor in the several groups are given by:

$$(53) \quad W_{i,t} = \alpha_{i,20} \frac{\partial Y_t^0}{\partial N_t^2} (N_{i,t}^2) (P_{t-1}) \quad i = 1, 2, 3$$

and

$$(54) \quad W_{4,t} = \left( \alpha_{4,20} \frac{\partial Y_t^0}{\partial N_t^2} (P_{t-1}) + \beta_{19} + \eta_j \right) N_{4,t}^2$$

Where the  $\alpha_{i,20}$  for  $i = 1, \dots, 4$  are proportionality factors

which will be discussed further below.

$W_{i,t}$  = labor income of  $i^{\text{th}}$  social group in  
constant currency

$\beta_{19}$  = a traditional constant component of  
income to Indians

$\eta_j$  = a random variable

In equations (53) and (54)  $P_{t-1}$  is used to show the effect of price level indicating a "price-stickiness" in the adjustment of wages.

The equation (53) is merely the standard returns to labor in a competitive economy modified by the  $\alpha_{i,20}$ . The equation (54) is a modified version of (53) where we assume that the income to the

Indians has three components, the first which is in proportion to marginal productivity, the second a constant in money terms determined from socio-economic considerations and the third a random variable determined by such things as weather and crop conditions.\*

#### 4.2.2 Gross Profits

Given the way we have written the production function for gross domestic product, the profit equations must include depreciation coverage as part of the returns to capital. In a competitive market we would have:

$$(55) \quad \Pi_{i,5} + D_{i,t} P_t = \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1} P_t \quad i, \dots, 6$$

where  $\alpha_{i,21}$  = factors of proportionality in return to capital.

#### 4.2.3 Income to the Institutional Factor

The third set of equations (56) given below represents the sources of payments to the institutional administrative factors. We assume that there are primarily seven sources, being respectively government investment and transfers, and private investment.  $\Pi_{5,t}$  includes all factor costs paid abroad including royalties and commissions.

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\* There is the difficult problem of imputing national income monetary returns to groups only partially in the money economy.

$Z_{i,t}$  = Income to the institutional factor supplied by the  $i^{\text{th}}$  social group.

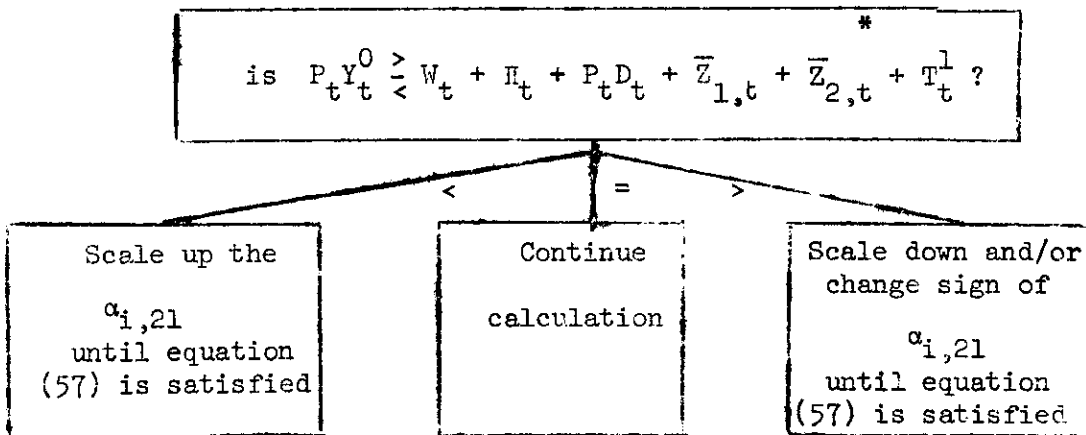
$$(56) \quad Z_{i,t} = \alpha_{i,22} G_{1,t} + \alpha_{i,23} (G_{2,t} + G_{3,t}) + \alpha_{i,24} R_t + \alpha_{i,26} I_{1,t} + \alpha_{i,27} I_{2,t} + \alpha_{i,28} I_{3,t} + \alpha_{i,29} I_{5,t} \quad i = 1, 2$$

We assume that income from this source only accrues to the commercial and agricultural whites. It indicates that they obtain a part of the funds spent by government, in transfers and in investment in their role as controllers of the bureaucratic structure of the country. A more socio-politically oriented model would have these shares depend upon the party in power and its major backing.

There remains the problem of consistency checking that the identity

$$(57) \quad P_t Y_t^0 = \sum_{i=1}^4 W_{i,t} + \sum_{i=1}^6 \Pi_{i,t} + P_t D_t + \sum Z_{i,t} + T_t^1 - \alpha_{i,24} R_t$$

must hold. This can be ensured in the manner shown in the flow diagram below.



\* For ease of solution, as we wish to exclude transfers we may define

$$\bar{Z}_{i,t} = Z_{i,t} - \alpha_{i,24} R_t$$

In order to guarantee that (57) holds we adjust the scale factors to the returns to capital. This implies that administrative costs and indirect business taxes, then labor, have the priority in being satisfied. After they have been met then the residual is paid to the other factor. A full explanation of the adjustment is given below.

#### 4.3. The Consistency of Income and Expenditure

A mixture of behavior and accounting equations has been written to describe the income and expenditures of all sectors and the interaction of foreign trade and financial flows. In order to bring consistency to the income and expenditure sides we introduce a gross correction in terms of a price level which is assumed to be endogenous to the system. The model given here for the whole economy is treated as open in the sense that foreign exchange levels are treated as exogenous.

Equations (50)

$$P_t Y_t^0 = Y_t - M_t^{2,1} + X_t^{2,1}$$

and (1)

$$Y_t = I_t + C_t + G_{2,t} + G_{3,t} + (X_t^1 - M_t^1) + (M_t^{2,1} - X_t^{2,1})$$

are sufficient to fix a price level when combined with (57).

$Y_t^0$  is determined directly from the Cobb-Douglas equation (51).  $M_t^{2,1}$  (see (64) and below) does not involve  $P_t$  or the necessity

to determine any set of variables simultaneously. This is not the case

with the identity (57). If we require this to hold we can do so by violating equation (55) and defining profits to be a residual. We make a further simplification that all profit sectors are hit equally by changes in economic conditions and the value of money.\* This is equivalent to assuming that there exists a variable  $x$  modifying the parameters  $\alpha_{i,21}$ . We may rewrite (57) as:

$$(I) \quad \left[ Y_t^0 - x \sum_{i=1}^6 \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1} \right] P_t = \sum_{i=1}^4 W_{i,t} + \sum_{i=1}^2 \bar{Z}_{i,t} + T_t^1$$

Substituting (50) into (1) we obtain:

$$(II) \quad P_t Y_t^0 = I_t + C_t + G_t^2 + G_t^3 + (X_t^1 - M_t^1)$$

We may solve (I) and (II) simultaneously in order to obtain the  $x$  and the  $P_t$ .

In equation (II) the  $C_t$  are dependent on current profits; hence involve both the  $x$  and the  $P_t$ , as is seen below:

$$C_{i,t} = \alpha_{i,1} N_{i,t} + \alpha_{i,2} Y_{i,t}^2 \quad \text{from (6)} \quad i = 1, \dots, 4$$

$$Y_{i,t}^2 = (1 - \alpha_{i,3}) Y_{i,t}^1 \quad \text{from (3 & 15)} \quad i = 1, \dots, 4$$

and (III)

$$Y_{i,t}^1 = x(1 - \beta_9)(\alpha_{i,25}) \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1} P_t + W_{i,t} + \bar{Z}_{i,t} + R_{i,t} + \alpha_{i,35} J_t$$

$i = 1, \dots, 4$

\*  
+ This should be modified, as it appears that governmental revenues are derived from a monopolistic sector and are more institutionally inflexible than the others.

Equation III and the relation based on equations (3 & 15) to link personal income with disposable income imply that transfers and payments to the administrative factor are also subject to income tax. Where the poor pay no income tax and the rich obtain little welfare payments the approximation is reasonable. The term  $(1 - \beta_9)$  represents profits tax paid before entrepreneurial income is obtained.

Summing over  $i = 1, \dots, 4$  to obtain  $C_t$  using III, 6, and 3 & 15 we may write II as:

$$\begin{aligned}
 \text{(IV)} \quad P_t Y_t^0 &= I_t + G_t^2 + G_t^3 + (X_t^1 - M_t^1)^* + \sum_{i=1}^4 \alpha_{i,1} N_{i,t} \\
 &+ \sum_{i=1}^4 \alpha_{i,2} (1 - \alpha_{i,3}) [W_{i,t} + \bar{Z}_{i,t} + R_{i,t}] \\
 &+ x(1 - \beta_9) \sum_{i=1}^4 \alpha_{i,2} (1 - \alpha_{i,3}) \alpha_{i,2} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1} P_t
 \end{aligned}$$

which is of the form

$$\text{(V)} \quad P_t Y_t^0 = A_1 + A_2 x P_t$$

In equation I we know that  $T_t^1 = \beta_7 Y_t^0 P_t + T_t^{1,2}$ . Hence

we may rewrite it as:

$$\text{(VI)} \quad P_t [(1 - \beta_7) Y_t^0 - A_3 x] = A_4 .$$

---

\* In different versions equations may be considered which imply more or less simultaneity. This must be checked in the following calculations.



Solving (V) and (VI) we obtain:

$$x = \frac{[A_4 - A_1(1 - \beta_7)]Y_t^0}{[A_2A_4 - A_1A_3]}$$

$$P_t = \frac{[A_1A_3 - A_2A_4]}{Y_t^0 [A_3 - (1 - \beta_7)A_2]} ; \quad \text{where:}$$

$$A_1 = I_t + G_{2,t} + G_{3,t} + (X_t^1 - M_t^1) + \sum_{i=1}^4 \alpha_{i,1} N_{i,t} \\ + \sum_{i=1}^4 \alpha_{i,2} (1 - \alpha_{i,3}) [W_{i,t} + \bar{Z}_{i,t} + R_{i,t}] .$$

$$A_2 = \sum_{i=1}^4 \alpha_{i,2} (1 - \beta_9) (1 - \alpha_{i,3}) \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1}$$

$$A_3 = \sum_{i=1}^6 \alpha_{i,21} \frac{\partial Y_t^0}{\partial K_{t-1}} K_{i,t-1}$$

$$A_4 = \sum_{i=1}^4 W_{i,t} + \sum_{i=1}^2 \bar{Z}_{i,t} + T_t^{1,2}$$

The mechanisms of monetary flows, banking regulations and price adjustment are probably among the more important features needing explicit description in an adequate development model. At this level of model construction only the barest formal consideration has been given to a rudimentary price adjustment mechanism.

#### 4.4 Retained Earnings

We assume that corporate enterprise is owned primarily by the commercial class and by foreign firms. In order to determine the income of the commercial class, it is necessary to distinguish those parts of profits which are paid out and those which are retained earnings.

$$(58) \quad \text{RET}_{i,t} = (1 - \beta_9) \Pi_{i,t} - \text{DIV}_{i,t} \quad i = 1,5 ; \text{ where:}$$

$$(59) \quad \text{DIV}_{i,t} = \alpha_{i,25} (1 - \beta_9) \Pi_{i,t}$$

Equation (59) states that paid out profits are a percentage of earnings.

#### 5. International Capital Flows and Transfers

Economic relations between a country and the rest of the world may be viewed in the context of imports and exports of goods and services, and imports and exports of capital. Imports and exports of goods and services have been discussed above; however, international capital movements have received little attention. These flows are a barometer reflecting basic conditions and problems within the country. Thus, an adverse trade balance will show up here and perhaps give an indication of whether devaluation, stricter exchange control or other such measures are needed. A developing social condition inimical to

the wealthier segment of the population may cause substantial short term capital movements out of the country which aggravate any balance of payments problems which might previously have existed.

The country's central bank, development banks, the World Bank, I.M.F., and other international monetary agencies together with the ministerio de hacienda form the dominant complex of institutions influencing capital flows. In any direct application of simulation for planning purposes, along with the internal price adjustment mechanism, they merit a more explicit treatment than is given here.

Capital inflows may be under any of three categories: transfers, long term flows and short term flows. The distinction between a transfer and another long term inflow is that no indebtedness or servicing is incurred on the former; they are essentially international gifts or accounting settlements and appear on current account.

A policy such as that of AID contains an intermix of transfer and other long term capital inflow where loans (even though they may be on very favorable terms) are dominant.

#### 5.1 Government Capital Flows and Transfers

$M_{t,6}^{2,2}$  = Net government borrowing from abroad

$M_{t,6}^{2,3}$  = Net transfer payments to government from abroad.

We treat both of these phenomena as exogenous, as they are primarily determined by policy and specific bargaining and do not lend themselves to easy behavioral equation modeling. (Equations 60 and 61) are omitted for the two exogenous series).

## 5.2 Long Term Private Capital Flow, Expatriated Earnings, and Transfers

$PM_t^{22}$  = long term net private capital inflow

$PM_{1,t}^{22}$  = direct investment (investment of foreign owned firms)

$PM_{2,t}^{22}$  = other long term capital (loans).

$$(62) \quad PM_t^{22} = PM_{1,t}^{22} + PM_{2,t}^{22}$$

This is merely an accounting equation to denote that long term private investment consists of the sum of the two components noted above. (In the program this disaggregation is not made owing to lack of information).

$$(63) \quad PM_t^{22} = (1 + \beta_{12}) I_{5,t}$$

Equation (63) states that long term private capital inflow will be somewhat greater than private foreign investment by working capital requirements.

$PM_t^{23}$  = Net private international transfers

This item is sufficiently small that it is assumed to be approximately zero.

$$(64) \quad X_t^{2,1} = \pi_{5,t} - T_{5,t}^2 + \beta_{13} \text{DET1}_t \text{FEXCH}_t$$

Factor income paid abroad includes expatriated earnings, interest and commissions.  $M_t^{21}$  is approximately zero.

### 5.3 Short Term Private Capital Flows

$BK_{1,t}$  = net foreign assets of banking system

$DBK_{1,t}$  = change in net foreign assets of banking system

$$(65) \quad DBK_{1,t} = BK_{1,t} - BK_{1,t-1}$$

$M_t^{2,4}$  = short term private capital inflow.

$BOP_t$  = Balance of payments surplus.

$$(66) \quad BOP_t = M_t^{2,4} + DBK_{1,t}$$

The balance of payments surplus equals short term private capital inflow plus the change in foreign assets of the banking system. We may define this as a residual as is shown in equation (66).

The categories used to divide up the assets and liabilities of the banking system are derived from those used in the monthly IMF publication, International Financial Statistics.<sup>5/</sup> Under the heading, "Monetary Survey," the IFS data gives the total liabilities of the banking system, excluding the development banks. In our application of the simulation to Ecuador we have included the development banks because the Central Bank of Ecuador in its own policy discussions defines Money Supply to include demand deposits at development banks. Another departure from the IFS system of accounts is that we have collapsed the government assets and liabilities of the banking system into one account, and the foreign assets and liabilities of the banking system into one account.

Banking System Assets:

$BK_{1,t}$  = Net foreign assets

$BK_{2,t}$  = Net claims on government

$BK_{3,t}$  = Claims on private sector

Banking System Liabilities:

$BK_{4,t}$  = Non-monetary liabilities to private sector

$BK_{5,t}$  = Money supply

$BK_{1,t}$  =  $BK_{1,t-1}$  +  $DBK_{1,t}$         where

$DBK_{1,t}$  = Change in net foreign assets of banking system

$$(69) \quad BK_{3,t} = BK_{3,t-1} + DBK_{3,t} \quad \text{where}$$

$DBK_{3,t}$  = change in credit to private sector during time  $t$  .

$$(70) \quad DBK_{3,t} = \beta_{20} + \beta_{21}I_{1,t}$$

As a first approximation, this states that the change in credit outstanding is a linear function of private investment. This is interdependent with the level of fixed investment not financed from abroad.

$$(71) \quad BK_{4,t} = BK_{4,t-1} + DBK_{4,t} \quad \text{where}$$

$DBK_{4,t}$  = change in non-monetary deposits of the private sector during time  $t$  .

$$(72) \quad DBK_{4,t} = \beta_{22} + \beta_{23}(S_{1,t} + S_{2,t} + S_{3,t}) \quad \text{where}$$

$S_{i,t}$  = personal saving of group  $i$  .

Equation (71) represents change in non-monetary deposits of private sector, and is determined by the level of personal saving.

$$(73) \quad BK_{5,t} = BK_{1,t} + BK_{2,t} + BK_{3,t} - BK_{4,t}$$

Equation (73) is the accounting relation for the money supply.

## 7. Concluding Comments

The previous sections have outlined a general aggregate model in which several features not usually common to macroeconomic models have been included. The following paper, Part III, is devoted to specializing the model to reflect the basic features of Ecuador, and then experimenting with simulations based on the specialized model.

There remain several further general comments before we turn to the operation of the model. They concern the relationship between this model, economic, political and social theory, and simulation as a form of economic history.<sup>6/</sup> Two other topics noted are the technical details of this model as a simulation, and the relationship between data processing and the input-output scheme adopted.

### 7.1. Description of the Simulation

The model constructed above is primarily macroeconomic with a four social sector disaggregation. Except for several logical switches which enable the program to execute policy or other decision changes the system is composed of a series of interlinked difference equations.

In general, difference equation models of economic processes are purely sequential; however, in this instance some of the relationships are simultaneous. This has made it necessary to solve a small simultaneous linear equation system in order to determine the price level.



Random variables do not play an important explicit conceptual role in the model, although they can be directly introduced in the input so that parameters, instead of being regarded as fixed, may be given variable values and outputs can be obtained in the form of average values together with standard deviations. This use of randomness aids in the exploration of the sensitivity of the system and plays a role when econometric measurement is attempted. Error terms may be added with ease to any equation. As is the case with all simulation, optimization is implicit rather than explicit.

The simulation here is a "representative unit," "fixed clock" model. This means it is so highly aggregated that any sector such as "the mestizos" is represented by a solitary behavior equation rather than a sample of behavioral units (as in the simulation of Orcutt)<sup>7/</sup>. The fixed clock feature of the model implies that all activities are updated at regular time intervals. There are no distributed lags to smooth the behavior of the behavioral units.

Among the natural extensions of this model would be to disaggregate production into at least agricultural and some industrial sectors. In some cases input-output matrices exist for the economy, thereby offering an available basis for disaggregation.

The current model contains approximately 135 equations, of which 65 are accounting; 60 are behavioral; and 10 are planning or policy relations which are for the most part treated as exogenous. Further interdisciplinary work might result in the specification of explicit relationships for variables currently left to be exogenous.

## 7.2 Simulation, Theory and Economic History

What is different in this approach from an input-output model? Is it not more confusing, arbitrary and ad hoc than a simple mathematically expressed and analyzed model? Does it not confuse economic, social, and other variables? What does it prove? How can one validate this type of model? Is it not of less value than a good verbal description?

The above questions are all relevant to work of this type and must be considered seriously.

The prime thesis behind this application of simulation is that it is a natural complementary approach to the currently more accepted methods in economic theory, econometrics, institutional studies, national income accounting, data processing, and planning. It has considerable potential as an integrative device. Its role in a data system is discussed in 7.3. Here the other aspects of the use of this approach are considered.

No technique or methodology can be of use itself when applied by those without a substantive knowledge and an understanding of basic theory. Hence "simulation" is no more an answer to the problems of the behavioral sciences than is "calculus." However, given a sufficient background, simulation offers a cheap and flexible method to augment the exploration of mathematical models of production growth and income. Sociological, political, and other hypotheses are often suggested. Before a heavy investment is made in research, theory, development, and

empirical work, it appears to be desirable to be able to obtain insights and estimations of the possible effects of various "guesstimates" on the structure of the environment.

A preliminary sensitivity analysis may be of considerable use in guiding the direction of deeper research. The model described here is offered in this spirit. It would be foolish to accept it, as it stands, as sufficiently detailed and trustworthy for detailed planning purposes.

By the judicious selection of parameter values at 0 or 1 many standard economic models can be obtained. If we believe in the Cobb-Douglas production function and competitive markets, limiting the allocation of product only to labor and capital according to marginal productivity, this can be conformed with in this model by cutting out the leakages to the institutional factor. If, however, we wish to check the effects of the conjecture that there are drains in administration we are in a position to do so.

Difference equation and other time dependent economic models are conjectural economic histories which up to some degree (not easy to define) may have some econometric validation. In general it appears that were it not for human inability to analyse the system, many socio-economic phenomena call for mathematical models in the form of non-linear simultaneous stochastic systems with distributed time lags and with logical switches and discontinuous functional forms.

We do not often construct models of the complexity noted above, because they appear to be so intractable both from the viewpoint of analysis and statistical validation. This leaves us, for the most part, with a theory for and statistical approach to simple mathematical models on the one hand and verbal description on the other. The type of model suggested here offers a third alternative. We are able to generate time series from structures of considerable complexity but may not have at this time adequate procedures for statistical validation; nevertheless, they provide a means for experimentation and "sweetening the intuition." Having experimented with alternative models and parameter values, the simulation may serve to aid in the selection of simpler models of greater realism and relevance than might otherwise have been constructed.

### 7.3 Output Display, Input Format and Data Banks

Data processing and data gathering are expensive occupations; but both are subject to vast external economies. It is my contention that more often than not the availability of data follows either theory or recognized need or both. The more a model constructed for the purposes of theory or planning is compatible with an ongoing data-gathering system, the greater are the economies in the obtaining of information, in influencing data-gathering procedures, and in communicating with others working on allied problems.

In Part III both the input format and output display associated with this model are discussed in detail. It must be restressed at this point that the output display is directly related to and cross-referenced with the economic data reporting system for AID devised at the Economic Growth Center at Yale. The selection of this output format was so that the simulation would be intimately related to a data processing system and a data bank and would have an output in a format that makes it easily available to and understandable by many.

The construction and use of large scale socio-economic models call for coordinated work by individuals with considerably varied training and skills. The costs of communication and coordination of effort are high. This approach is directed at least partially towards lessening them.

FOOTNOTES

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7. Orcutt, G. H., M. Greenberger, J. Korbel, and A. H. Rivlin, Microanalysis of Socioeconomic Systems - A Simulation Study, New York: Harpers, 1961.